The British Sundial Society

BULLETIN

VOLUME 13 (ii) JUNE 2001

Front Cover: *Wren Dial. All Souls' College, Oxford (Photo: M. Hewitt)*

Back Cover: *Horizontal dial, Mary Washington's House, Fredericksburg, Virginia, USA: Lat: 38° 20'*

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OF THE BRITISH SUNDIAL SOCIETY

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EDITORIAL

In this issue of the Bulletin, we commend to readers especially the article on the Double Horizontal Sundial: a handsome and intricate design, but baffling to anyone coming across an example for the first time. This design flourished over a period of less than 100 years, from about 1630 until the end of the 17th century. Michael Lowne's thorough and comprehensive article not only explains how to read such a dial but also lists all examples so far discovered. His article will certainly be used as a reference for many years to come. But one cannot say that this is the last word on the subject because more early examples may yet appear. Also the author tells us how to make one, and mentions modern examples made by himself and by John Davis. So perhaps there will be a spate of 21st century double-horizontals.

Since the last issue of the Bulletin, this year's Annual Conference and AGM have taken place, over an enjoyable weekend in late April. York (College of Ripon and York) made a delightful venue. There was plenty of interest in the talks, numerous dials on display, a bus tour of dials in the Vale of York, and a walkabout to see the City's dials. A full and illustrated report will be given in the September Bulletin.

THE DESIGN AND CHARACTERISTICS OF THE DOUBLE-HORIZONTAL SUNDIAL

MICHAEL LOWNE

INTRODUCTION

The seventeenth century was a time of rapid development in many branches of science, particularly those of astronomy and mathematics. Pre-eminent in this latter field was the English mathematician William Oughtred (1575-1660)[1]. Born at Eton, he was educated at Eton College and at King's College, Cambridge. After ordination as a priest in about 1603 he held the livings at Shalford and at Albury, both in Surrey, only occasionally visiting London. He took mathematical pupils and maintained correspondence with other leading mathematicians of his day which together with publications in Latin and English established his reputation at home and abroad. An engraved printing plate of a portrait of Oughtred has recently come to light[2]. His contributions to the art of gnomonics included the design of the 'horizontal instrument' and the 'double-horizontal sundial'.

COMPARISON OF THE TWO INSTRUMENTS

It is necessary to distinguish between the two related but separate instruments. The first version (the horizontal instrument) was devised early in the seventeenth century. It is portable and consists of a plan of the sky on a horizontal plane, delineated with hour circles, parallels of declination and ecliptic arcs with date scales enabling the sun's declination to be found for any day. There is a scale of altitudes around the rim and a centrally-pivoted alidade also carrying an altitude scale. In use the instrument is suspended vertically by a ring at the top and held edge-on to the sun so that the shadow of a central pin falls on the outer altitude scale. The reading from this is then transferred to the alidade which is rotated until the altitude coincides with the declination of the sun for that day. The time can then be read at this coincidence point and the times of sunrise and sunset can also be determined. Instrument makers such as Elias Allen became aware of the instrument and made fine examples, although no printed information was available until 1632[3] when a pupil of Oughtred, William Forster, translated and published Oughtred's Latin manuscript which also included a description of his 'Circles of Proportion', a circular slide-rule.

The double-horizontal sundial is a fixed-dial development of the portable horizontal instrument. There are two sets of graduations on the baseplate, the more usual variety with time-marks corresponding to an inclined polar gnomon, and another set with a central vertical gnomon and graduations which are a plan of the sky in the same format as the earlier instrument. Having a vertical gnomon this part of the dial operates from the azimuth of the sun, that is to say the bearing of the point where a vertical line through the sun meets the horizon. The essential difference between the two instruments is therefore that the one operates from the altitude and the other from the azimuth of the sun. Both are made for a specific latitude and require knowledge of the sun's declination as the third parameter. Descriptions of the double-horizontal dial and its uses were given by Oughtred in pamphlets issued in London in 1636[4] and later editions.

Studies of the dial by A J Turner[5] and F W Sawyer[6] have been of assistance in the preparation of this article. Turner gives details of an acrimonious dispute between Oughtred and Richard Delamain, also a mathematician, who claimed priority in the invention of the circles of proportion and the horizontal instrument. A reprint of Oughtred's 1636 description and uses is provided by Sawyer.

It appears that the first double-horizontal dials were made by Elias Allen but they were soon produced by other instrument makers. Few of the dials are dated but from the known dates of the makers it seems that the dials were not in vogue for very long, the last known example being made by Benjamin Scott in about 1713.

DESCRIPTION OF A DIAL

The surviving double-horizontal dials are now three hundred or more years old and where they have been left *in situ* are quite possibly patinated or eroded and difficult to decipher[7]. One well-preserved example is a very fine dial by John Seller which is now in a private collection. Seller was not only an instrument maker: he was a cartographer and held the appointment of 'Hydrographer to the King' over three or four reigns. Figure 1 is a view of his dial (which is made for latitude 51½°N) showing the inclined polar gnomon and tucked underneath it the shorter vertical gnomon. The long polar gnomon is necessary to ensure that the shadow reaches the outer graduations when the sun is high in the sky. The fillet around the gnomon base would interfere with the operation of both dials at low solar altitudes and suggests a later repair or replacement. A face-on drawing of the dial-plate is shown in Figure 2: here the Roman numerals and time graduations (divided to five-

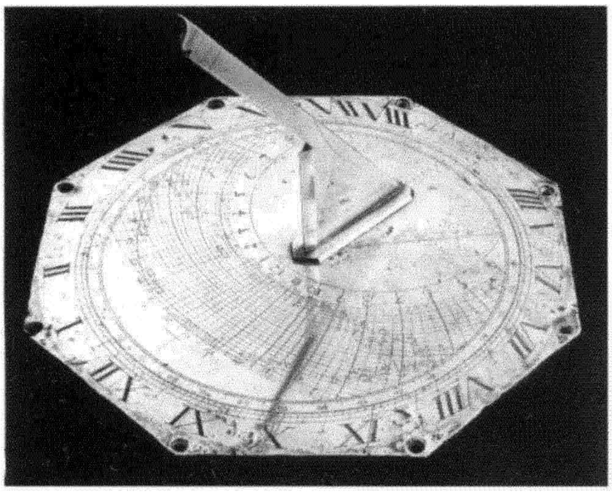

Fig.1. Double-Horizontal dial by John Seller

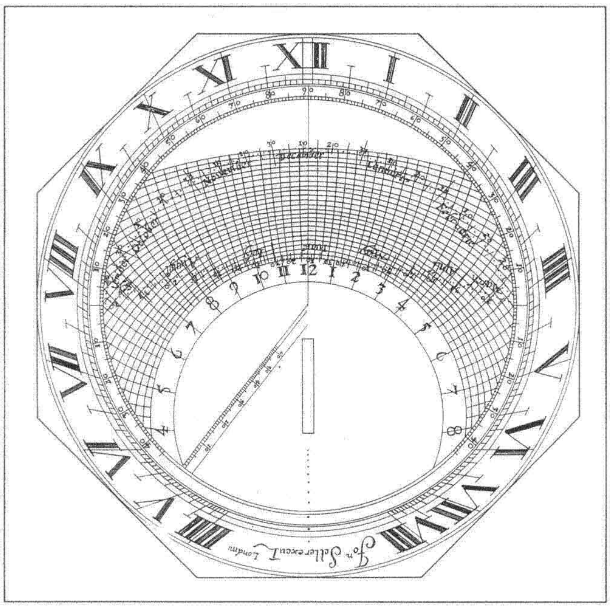

Fig.2. Dial-plate of the Seller dial

minute intervals) relating to the polar gnomon are seen around the periphery. Inside these are the graduations of the azimuth dial which enable a number of astronomical quantities to be derived from the shadow of the vertical gnomon. The shadow-casting part of this gnomon is chamfered to a sharp vertex: there is then no need for a discontinuity in the graduations at noon to allow for the gnomon thickness as with the polar dial. However, the azimuth dial reading is limited by the vertex angle of the gnomon. In the Seller dial (and others) this is about 30°, so that if the sun is within 15° of the meridian the shadow will be cast, not by the vertex, but by one or other of the edges of the V, giving incorrect readings. This will occur between about 11am-1pm in midwinter and 11.30am-12.30pm in midsummer, in the latitude of the British Isles.

THE AZIMUTH DIAL

The shadow of the vertical gnomon on the dial face is diametrically opposed to the azimuth of the sun: the dial plan is therefore turned through 180° with respect to the sky. Immediately inside the polar dial time graduations is a circle centred on the foot of the vertical gnomon. This represents the horizon and is divided to degree intervals of azimuth and labelled every 10°. The zero points are to east and west and the numbers increase until they meet at 90° at the south point. North of east and west they again increase but only to 40° on each side. These are the northerly limits of sunrise and sunset at the summer solstice and further graduation is unnecessary.

Running across the dial from east to west is an array of curved arcs with some thicker than others. These represent the path of the sun across the sky depending on its declination, the angular distance north or south of the celestial equator. They are not labelled but it is easy to see that the central thick line is the equator and others are spaced at two-degree intervals (only $1\frac{1}{2}^\circ$ for the outer interval from 22° to $23\frac{1}{2}^\circ$) with thicker lines at 10° and 20° north and south. The thick lines are not shown on Figure 2.

Crossing the declination lines are others labelled with Arabic numerals from 4 in the east through 12 in the north-south line to 8 in the west. These are hourly time-lines and each hour is divided into quarters. Some dials have declination arcs for every degree: the dial by Thomas Tuttell shown in Figure 3 is one example. The time scale on the Tompion dial at Hampton Court[8] is divided to five-minute intervals.

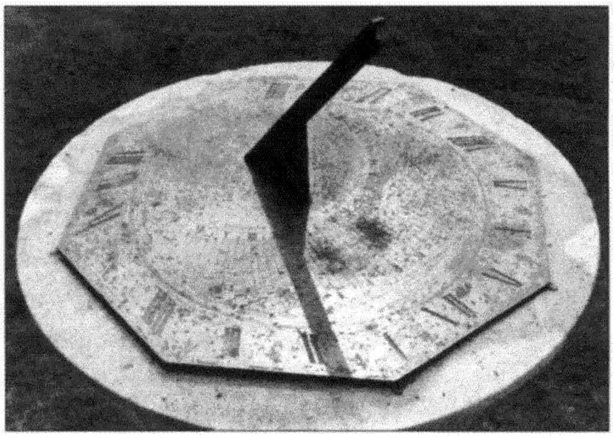

Fig.3. A dial by Thomas Tuttell

Two arcs meet the azimuth circle at the east and west points and curve round to touch the declination lines at the extremes of $\pm 23\frac{1}{2}^\circ$ on the north-south line. These show the annual path of the sun through the sky (the ecliptic) inclined at $23\frac{1}{2}^\circ$ to the equator, the tilt of the Earth's axis of rotation relative to the orbit (the obliquity). They carry a scale of dates, divided to individual days and labelled for the 10th, 20th, and last day of the month. The declination of the sun for every day of the year is thus indicated, but individual days between the 5-day markers are shown only schematically on Figure 2. The solstices (sun at $23\frac{1}{2}$°) are shown as June 11 and December 11, and the equinoxes (sun at 0°) as March 11 and September 13, ten days earlier than our present reckoning. When this dial was made in the late seventeenth century Britain was still using the Julian calendar, at that time ten days behind the reformed Gregorian calendar we now use.

A diagonal scale running from the gnomon point to the NE horizon is an almucantar for determining the altitude of the sun. It is divided to degrees and labelled every 10°.

THE USE OF THE DIAL

Much astronomical information can be found from this construction and the shadow of the vertical gnomon. By reading the azimuth scale where it is met by the shadow (or its extension if it doesn't reach that far) the sun's azimuth can be determined. If the date is known the time can be found by locating the date on the ecliptic scale, noting the corresponding declination (probably between two lines) and following that declination round until it intersects the gnomon shadow. The position on the hour scale of this intersection shows the time. In a reverse procedure, the declination and approximate date can be found be taking the time from the outer dial, transferring this to the corresponding inner dial time-line and noting the declination where this crosses the gnomon shadow. Following this back to the ecliptic gives the date. This method of finding the date is not very sensitive: the maximum rate of change of the sun's declination (at the equinoxes) is only 0.4°/day. At the solstices the declination changes very slowly and remains effectively at the same value for several days at a time. Some dials, notably the late example by Benjamin Scott, have the scale of dates at the ends of the declination arcs. A few dials do not carry a date scale: for these it is necessary to use the time derived from the outer dial as explained above to find the declination.

Having found the declination of the sun, the line can be followed to the horizon circle at east or west side. This will give the times of sunrise and sunset by reading the time graduations at those points. The azimuth at rising and setting can also be found from the horizon circle. The times of sunrise and set can of course be found for any day of the year, not just the current date, by following the appropriate declination line to the horizon.

The altitude of the sun is found by setting one point of dividers upon the derived position of the sun at the declination-gnomon shadow intersection and the other on the foot of the vertical gnomon. Transferring this distance to the almucantar scale enables the sun's altitude to be read off.

These properties are demonstrated by the simplified drawing of the south-west quadrant of the azimuth dial in Figure 4, with just sufficient detail to show the principle. A diagonal line represents a gnomon shadow which indicates a solar azimuth of 50° south of west. Suppose the date is February 13: this day-mark on the ecliptic curve gives the declination as –10°. Following this line to its intersection with the shadow shows the time to be just after 2.30pm, say 2.32. In Table 1 these and other quantities derived from the dial by the above methods are compared with values calculated for that azimuth and date.

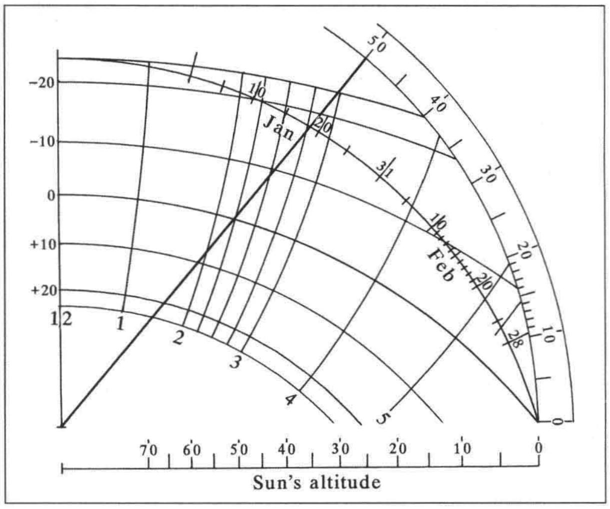

Fig.4. To illustrate the use of the azimuth dial

TABLE 1

Quantity	Derived from dial	Calculated
Declination	-10°	-9.9°
Time	2.32pm	2.31pm
Sunset	5.9pm	5.10pm
-at azimuth	16°S of W	16.0°S of W
Sun's altitude	20°	20.5°

As is traditional in gnomonics, the effects of atmospheric refraction are ignored in the time and azimuth of sunset. The agreements between the calculated values and those derived from the dial are most satisfactory!

In the 1636 description Oughtred gives no less than twenty possible uses of the dial, although some of these are rather repetitious. In addition to those already mentioned, it is possible by use of the azimuth scale to determine the declining angle of a wall, either with the dial unmounted or permanently fixed. In the latter case a horizontal board is used with a line drawn perpendicular to the wall. The shadow of a plumb-line as it crosses this line is marked off. Then, says Oughtred, "run instantly to the dial" and take the reading of the sun's azimuth. This angle, if drawn relative to the marked position of the plumb-line shadow, shows the north-south line. The angle between this and the perpendicular to the wall gives the declining angle.

By imagining the dial to represent the hemisphere of the sky which is below the horizon, the duration of twilight and the depression of the sun can be found.

Another quantity (apparently not envisaged by Oughtred) which can be deduced is the right ascension (RA) of the sun, the angular distance along the celestial equator (measured in time) from the March equinox to the hour circle through the sun. This can be obtained by ignoring the time figures and counting the hours and minutes forward along the equator from the equinoctial points to the hour line containing the required date, starting the count at 0 hours at the March equinox and 12 hours at that of September. As the sidereal time is defined by the RA which is due south on the meridian, the sidereal time at apparent noon is equal to the sun's RA. Adding 12 hours to this (strictly 12 hours 2 minutes) will give the sidereal time at midnight. A dial by Elias Allen now in the Science Museum and illustrated by Turner[5] has subsidiary small figures along the 0° declination arc to facilitate determining the RA of the sun in this way. It also gives the RA of twelve bright stars to the nearest quarter-hour These include 'Great Dog', 'Lion's heart', 'Bull's eye' and 'The Goate', respectively Sirius, Regulus, Aldebaran and Capella. There is 'A Moone Dial' consisting of small Arabic numerals just inside the Roman numerals of the polar dial and a table of ages of the moon and corresponding hours to correct the dial reading. The presence of the moon dial implies that the star table is also intended as a means of telling the time at night. I am not suggesting that the wakeful owner should venture out in his nightshirt to consult his dial by candle-light; it would be simple to memorise the midnight sidereal time and the RA of a few stars which transit in the hours of darkness at that season. By observing the position relative to the meridian of a star of known RA an estimate of the sidereal time can be made. The difference between this and the midnight sidereal time will give an approximate figure for the solar time.

Other dials, notably some of the large instruments made by Henry Wynne, also carry moon-dials and the RA of some stars.

THE SELF-SETTING PROPERTY

The combination of polar and azimuth dials implies that the instrument is self-setting: by rotating it in a horizontal plane until both dials show the same time it will in principle be correctly oriented in the meridian. Although Oughtred makes much use of this property he does not mention the limitations of determining the orientation in this way. The crucial factor is the error in the orientation which will be caused by a small difference between the times shown by the two dials. From practical tests I find that the mutual agreement of the time readings cannot be judged to better than a minute. Even such a small difference will introduce an orientation error whose extent is strongly dependent on the hour-angle of the sun at the time of the operation and is also affected by the sun's declination and the latitude. Figure 5 shows the relation between the hour-angle of the sun and the orientation error, the angle through which the dial can be rotated to either side of the true position before a difference of one minute between the dial readings can be detected. It is drawn for latitude 50°N and two declinations, +20° and −20°. Looking at the +20° (summer) line, the orientation error for one minute difference between the dial readings cannot be less than about ½° and then only if the sun is at least four hours off the meridian. At smaller hour angles the error builds up very rapidly and it would be useless to try to obtain an accurate setting if the sun is within about 2½ hours of meridian passage. In winter (-20°) it is hardly possible to obtain an satisfactory orientation at all. The latitude dependence is such that at 58° the error is about 50% greater than shown in Figure 5 but is only about half as much at 40°.

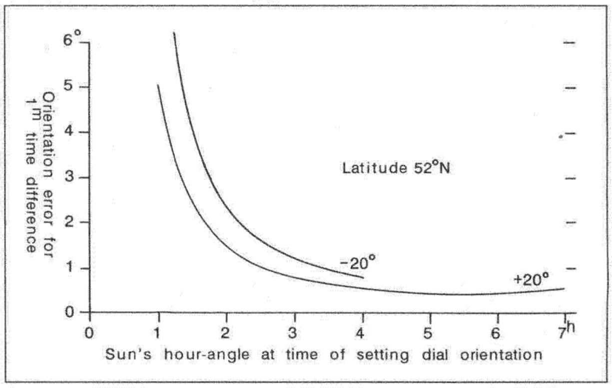

Fig.5. The orientation error for one minute time difference

Any error in the orientation will of course have an effect upon the time-keeping of the dial. Broadly, an error of one degree in the orientation will cause an error of four minutes in the indicated time. Even at the most favourable time of setting the orientation the time error caused by one minute difference between the dials will be about 2 minutes. The error rises rapidly under unfavourable conditions and the dials could be as much as twenty minutes wrong if orientation is attempted with the sun near the meridian. Matters can be improved by rotating the dial first in one direction and then the other to find and mark the two positions where a time difference can be detected. The mid-point of the two marks will provide a fair indication of the true orientation if the sun is not too close to meridian passage.

THE MAP OF THE SKY

In common with all map-makers[1] Oughtred was faced with the problem of depicting part of a sphere (in this case the complete visible hemisphere of the sky) on a plane surface. The resulting flat map is termed the projection of the sphere and many types of projection have been devised. All involve some distortion in representing a sphere on a plane, and it is a question of selecting the best projection for the particular purpose. The one used by Oughtred is known as the stereographic projection (Greek *stereos*, solid, *graphein*, to write). In the 17th century this was much used for terrestrial and celestial maps, such as those by Cellarius[9] sometimes reproduced as calendar illustrations. It suffers from distortion and large variation of scale, but for the present purposes these defects are not significant and the projection has other properties which make it ideal for this application. One of the useful properties is that circles on the sky project as circles on the plane, except for great circles passing through the origin, which project as straight lines. This contrasts with the gnomonic projection used in polar-gnomon plane dials, in which great circles project as straight lines and small circles appear as conic sections. (Great circles are those whose plane passes through the centre of the sphere: all other circles are small circles. For example, on the Earth meridians of longitude and the equator are great circles and parallels of latitude are small circles.) Another useful property of the stereographic projection is that angles on the sky are reproduced on the map, in the sense that the tangents to the arcs of projected lines at their intersection meet at the same angle as the corresponding arcs on the sky: this is of use in a geometrical delineation of the dial. Except in the case of the horizon the centres of the circles on the map do not coincide with the corresponding centres on the sky and the necessary centres and radii to draw them must be found by calculation or geometric construction. In Figures 1 and 2 the row of small pits on the line of the gnomon are some of the centres from which the declination arcs were struck.

The principle of the stereographic projection applied to the case of a horizontal dial-plate is shown in Figure 6. The circle NZSO represents the meridian with Z the zenith and 0 the opposite point, the nadir. NESW is the dial plate in the plane of the horizon and ZCO is the vertical through its centre. A position on the sky is shown at P and ZPF is the vertical arc through P meeting the dial plane at F. The angle NCF is the azimith (A) of P from north. The nadir 0 is the origin of the projection and a line joining P0 cuts the plane NESW at Q which is called the stereographic projection of P. As Q lies in the line CF, the azimuth of P from the north point is exactly reproduced on the dial by angle NCQ.

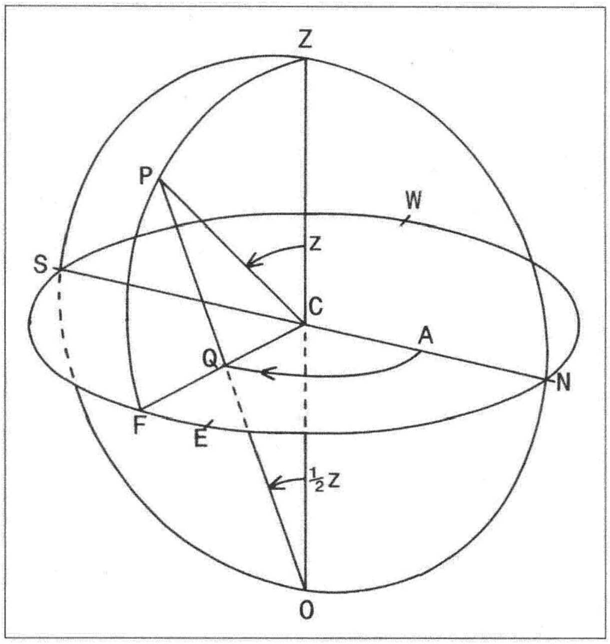

Fig.6. The principle of the stereographic projection

By joining CP, the angle ZCP is the angular distance of P from Z, called the zenith distance (z) of P. The triangle PCO is isosceles (PC=OC, both radii of the circle) so the angle COQ is $\frac{1}{2}z$. Triangle OCQ is right-angled at C, therefore the distance CQ is proportional to $\tan\frac{1}{2}z$. At the horizon $Z=90°$ and $\frac{1}{2}z=45°$ whose tangent is 1, so if R is the chosen radius CF of the horizon circle on the dial the distance CQ is $R\tan\frac{1}{2}z$.

In the following formulae and those of the next section, the symbols used are:

Latitude	ϕ
Sun's hour-angle	h
Sun's declination	δ
Obliquity of the ecliptic (23.44°)	ε
Radius of the horizon circle on the dial	R
Radius required to draw a projected circle	r

The rectangular coordinates x and y on the projection are referred to the meridian and the prime vertical, the great circle which passes through the zenith from the east-west points of the horizon. Both circles project on to the dial as straight lines as shown in Figure 7. In astronomical usage, x increases to the west and y increases to the north so, remembering that the dial plan is rotated by 180° and taking the centre of the dial as the zero-point, the signs of x and y appear on the dial as in Figure 7. (Sawyer[6] uses the opposite sign convention for y.)

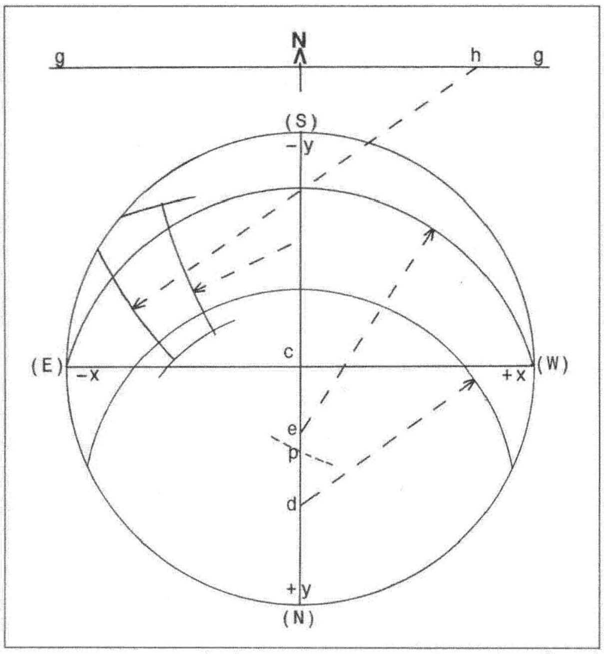

Fig.7. Delineating the azimuth dial

Any point on the sky (defined by its hour-angle and declination and the latitude), can be plotted from the general formulae:

$$x = \frac{R(\cos\delta.\ \sin h)}{(1+\sin\phi.\ \sin\delta + \cos\phi.\ \cos\delta.\ \cos h)} \qquad 1.$$

$$y = \frac{R(\cos\phi.\ \sin\delta - \sin\phi.\ \cos\delta.\ \cos h)}{(1+\sin\phi.\ \sin\delta + \cos\phi.\ \cos\delta\ .\cos h)} \qquad 2.$$

DELINEATION OF THE STEREOGRAPHIC PROJECTION

An interesting challenge to a present-day dial-maker would be to build a modern version of the double-horizontal dial. My own effort (somewhat simplified and made only in chipboard and plywood) is shown in Figure 8. The early dial-makers most probably used geometrical methods for the delineation but in these days of computers (or even pocket calculators) it is a simple matter to calculate the necessary details.

The arcs of the projected declinations and the two ecliptic curves are symmetrical on either side of noon, so that their centres lie on the meridian (x=0) line. In Figure 7 a declination arc is shown centred at d and one of the ecliptic arcs centred at e.

The centre of a declination arc is given by:

$$y = R\cos\phi/(\sin\phi+\sin\delta) \qquad 3.$$

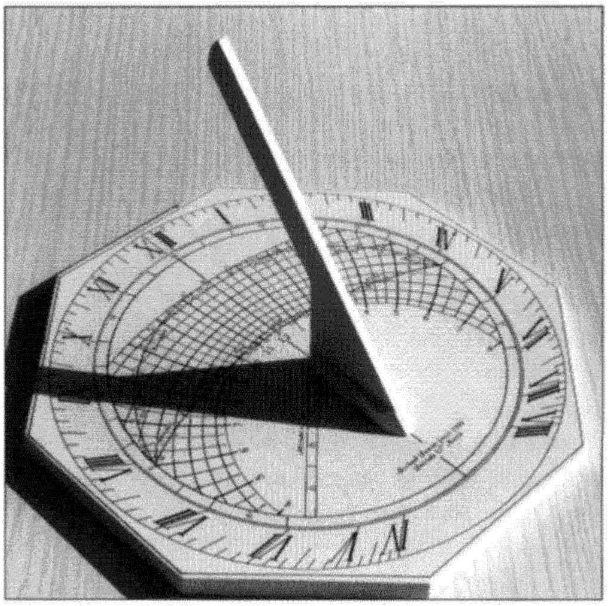

Fig.8. A modern double-horizontal dial by the author

The radius required to draw the arc is:

$$r = R\cos\delta/(\sin\phi+\sin\delta) \qquad 4.$$

The two ecliptic arcs require different centres and radii. For the March-June-September arc:

$$y = R/\tan(\phi-\varepsilon) \qquad 5.$$

$$r = R/\sin(\phi-\varepsilon) \qquad 6.$$

and for the September-December-March arc:

$y = R/\tan(\phi-\varepsilon)$ see errata on next page 7.

$r = R/\sin(\phi-\varepsilon)$ see errata on next page 8.

The centres of the hour lines (as at h) lie on a straight line g-g which is parallel to the x-axis. For this:

$$y = -R\tan\phi \qquad 9.$$

The centres and radii of the arcs are given by:

$$x = -R/(\cos\phi.\tan h) \qquad 10.$$

$$r = R/(\cos\phi.\sin h) \qquad 11.$$

Lines which are separated by 12 hours have the same centres and radii. The hour lines need only be drawn over the range covered by the declination arcs between +23.5° and –23.5°, or down to the horizon if –23.5° is below the horizon at that point, as can be seen on Figures 1 and 2 and indicated on Figure 7. It will be found that lines close to the meridian are nearly straight, with centres at considerable

ERRATA

In *BSS.Bull.* 13: **'Design and Characteristics of the Double Horizontal Sundial'**

p.52: Equation 7 should read: $y = R/\tan(\phi+\varepsilon)$

Equation 8 should read: $r = R/\sin(\phi+\varepsilon)$

p.53: Equation 15 should read: $x = R\cos L/(1-\cos(\phi+\varepsilon).\sin L)$

Equation 16 should read: $y = R\sin(\phi+\varepsilon).\sin L/(1-\cos(\phi+\varepsilon).\sin L)$

distances from the dial centre-line. If the radius required is more than can conveniently be handled by compasses or trammels, it is possible to calculate the x,y positions at intervals by the general formulae (1,2), plot the resulting points and join them with a suitable curve.

Some of the formulae for the radius r may give a negative value: this may be ignored and the absolute value taken.

In delineating the dial, various checks can be made. The arc for the equator (δ=0°) should pass through the east and west points of the horizon, as will the two ecliptic arcs, which should also touch the limits of declination (23.44°) on the meridian. Although it is not necessary to extend the hour lines beyond the range of the declination arcs, they should pass through the projected pole of the sky which is on the meridian and at $y=\tan\frac{1}{2}(90-\phi)$, p in Figure 7. The arc for 6 hours hour-angle (h=90°) is centred at x=0 and should pass through the east and west points of the horizon.

It remains now to put the scale of dates on the ecliptic arcs. The distance along the ecliptic traversed by the sun during 24 hours varies, due to the elliptical nature of the Earth's orbit around the sun and the resulting variations of orbital velocity. The average daily motion of the Sun is 59 arc-minutes, but when the Earth is closest to the Sun (at perihelion) in early January it is 61'/day and at its furthest (aphelion) in early July it is 57'/day. If the mean and true sun are taken as together at perihelion, three months later the sun is 1.92° (or nearly two days' motion) ahead of the position it would occupy if the Earth's orbit were circular. The sun then drops back until at aphelion mean and true positions again coincide. After three months the sun is 1.92° behind the circular motion position.

The x,y coordinates of the sun on the dial for each day can be derived from the true longitude of the sun, its angular distance along the ecliptic from the 'first point of Aries' at longitude 0° where the ecliptic intersects the equator at the March equinox. Throughout the leap-year cycle there are small variations in the daily longitudes, but for the present purpose these are disregarded and average values taken. It is convenient to use the midday values of the longitude which is calculated by applying a correction for the elliptic motion to the longitudes found from the average daily motion of the sun of 0.9856°/day (=360°/365.25 days). The longitude (averaged over the leap-year cycle and adjusted to allow for the leap-year differences) on December 31 at midday is 280.1° and at the present time the Earth is at perihelion on January 3. The sun's longitude L is given by:

$$L = 280{\cdot}1+0{\cdot}9856d+1{\cdot}92\sin(0{\cdot}9856(d-3)) \qquad 12.$$

where d is the day of the year: January 1=day 1, February 1=day 32 and so on. February 29 is ignored.

The x,y coordinates of the daily points are given by:

from March 21 to September 22 (L 0° to 180°)

$$x = R\cos L/(1+\cos(\phi-\varepsilon).\sin L) \qquad 13.$$

$$y = -R\sin(\phi-\varepsilon).\sin L/(1+\cos(\phi-\varepsilon).\sin L) \qquad 14.$$

and from September 23 to March 20 (L 180° to 360°)

see errata on previous page

$$x = R\cos L/(1-\cos(\phi-\varepsilon).\sin L) \qquad 15.$$

see errata on previous page

$$y = R\sin(\phi-\varepsilon).\sin L)(1-\cos(\phi-\varepsilon).\sin L) \qquad 16.$$

The daily points for the sun's position should of course lie along the pre-drawn curves for the ecliptic.

The almucantar operates from the zenith distance (z) of the sun but is graduated for the altitude (a): z = (90-a). Radial distances from the foot of the gnomon are given by:

$$r = R\tan\tfrac{1}{2}(90-a) \qquad 17.$$

The height of the vertical gnomon should be not less than half the radius of the horizon circle. This constrains the placing of the root of the polar gnomon and at low latitudes may result in an excessive distance from the dial centre.

DELINEATION BY GEOMETRICAL METHODS

As mentioned earlier, it is possible to delineate the dial by geometrical construction and it seems most probable that the seventeenth century dials were made that way. Sawyer[6] has given full details and it is not proposed to repeat the explanation here. Briefly, the principle is that the projected arcs, if drawn as complete circles, would intersect the meridian line in two points, generally one above and one below the horizon. The centre of the arcs must lie midway between the projections of these two points and the radius is half their separation. The line of the centres of the hour arcs is at the mid-point of the projections of the north and south poles and the centres are found by drawing lines from the pole at the appropriate hour-angles to meet the line of centres. From these points the arc radius is the distance to the pole. A major difficulty with the geometric method is that some of the projected points lie at considerable distances from the centre of the dial, necessitating the use of a large setting-out table. Such an item would have been common in an early instrument-maker's workshop.

How the scale of dates was inserted on the ecliptic is not known: one method would have been to plot the tabulated right ascension of the sun for each day along the ecliptic arc using the hour-lines, the reverse of the procedure mentioned earlier.

THE EXISTING DIALS

Table 2 is a list of known dials, listed by maker and showing the date where this is known (or in some cases an inspired guess), the material and dimensions, the present location and other brief details. It cannot be taken as complete: early sundials are still being discovered and other double-horizontal dials may be among them.

Table 2

Elias Allen, (fl 1606-1654):			
Undated	Brass, 12" octagonal	Museum of the History of Science, Oxford	
Undated	Brass, 12" octagonal	National Maritime Museum, Greenwich	
Undated	Brass, 12" octagonal	Science Museum, London. 'latitude 51½°N	a.
Bacon (? maker or owner)			
Undated	Brass, 13" square	Not known. Lat.51°56'. Eq. of time table	b.
Hilkiah Bedford, (fl.c 1656-1680, d1689):			
1668	No details	St Andrews.	
Jacob Clark:			
c1680	Brass, octagonal	National Museum of Scotland Edinburgh	
c1700	Brass, octagonal	National Museum & Gallery Merseyside	
Daniel Delander (1678-1733):			
Undated	Bronze, 24" diam	Northamptonshire. SR3607 Lat. 51½°N	
Ralph Greatorex, (1625-1712):			
Undated	Brass, square	Private collection	
Stephen Gray:			
1699	Brass, 12" square	Kent. SR4123	
Benjamin Scott (fl 1712-1751):			
c1713	Brass, 20½" diam	Paris	
John Seller (fl 1658-97):			
c1680	Brass, 8½" octagonal	Private collection. SR3121 Lat. 51½°N, Figures 1 & 2.	
Henry Sutton (working 1649, d1665):			
1658	Brass	Not known	
1659	Square	Not known. Broken gnomon	c.
Thomas Tompion (1638-1713):			
1690	Brass, 20½" diam	Hampton Court. SR2119	
Thomas Tuttell (working 1693, d1702):			
c1700	Brass, 15" diam	Kent, Figure 3. SR3539	

Henry Wynne, (fl 1654-1709):			
1682	Brass, 30½" diam	English Heritage SR2125	d.
c1690	Brass	Grafton Hall	
c1690	Brass	Powys Castle	
c1690	Brass, 36" diam	Norfolk. Lunar table, equation of time.	
c1690	Brass, 27" diam	Not known	e.
1692	Brass, 33" diam	Dumfriesshire. Lat 56°N. Spiral moon dial. SR0897	f.
1695	Brass	Not known	
Undated	Brass, circular	London	
Anon:			
mid 17c	Brass, 13" octagonal	Illinois	
Undated	Brass, octagonal	Kent, SROOO1	g.
Undated	Bronze, 14½" square	Somerset, SR3949 Gnomon not original.	
MODERN DIALS:			
Michael Lowne:			
1999	Chipboard and plywood 10½" octagonal	Sussex. Lat.51°N. Figure 8	
John Davis:			
2000	Brass, 12" octagonal	Suffolk. Lat.52°4'N.	

NOTES TO TABLE 2:

a. This is the dial described in the text, with moon dial and star positions.

b. The date scale is on the horizon at the ends of the dec. lines. Advertised for sale in Spring 2000 for $4950.

c. Sold at auction for £920[10].

d. A replica of this dial has been made for use at Wrest Park.

e. Sold at auction for £11500 in 1990.

f. A recent photograph has appeared in a BSS *Bulletin*[11].

g. This dial has the stereographic plan to the south of the gnomon. It is apparently read from alidades which can be pointed towards the sun, thereby duplicating the function of the shadow of a vertical gnomon.

The notation SR is the entry number in *The Sundial Register*, 2000[12].

More than half the 17th and 18th century makers listed are represented by only one dial. It has been suggested that possibly some of these solitary examples were made by instrument-makers' apprentices at the conclusion of their indentures to demonstrate their capabilities.

ACKNOWLEDGEMENTS

The photograph and drawing of the Seller dial in Figures 1 and 2 were kindly provided by the owner of the dial. The major part of the list of surviving dials was supplied by Mr Christopher St J H Daniel: other information was provided by Mr J Moir, Mr M Kenn and Dr J R Davis. I am indebted also to Mr Daniel for the photograph of Figure 3 and for much technical help and encouragement. Miss R J Wilson provided biographical details of some dial makers.

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A POCKET GNOMON PROTRACTOR

JOHN DAVIS

Most dial hunters have a precision protractor which they use to measure gnomon angles when out "on safari". However, these tend to be bulky and expensive instruments, so they are not something which is carried around all the time. I wanted something flexible that I could slip into a wallet or the cover of a notebook, so I set about making one.

On many horizontal dials, the base of the gnomon is buttressed to give it strength, with the result that it is very difficult to stand a simple "schoolroom" protractor alongside the gnomon. The area of the dialplate to the south of the gnomon is usually flat and unobstructed, though, making it easy to measure the complement of the gnomon angle. Also, the range of angles that needs to be measured in the UK is very restricted, leading to a compressed scale, much of it unused. I realised that it was possible to overcome both these problems with some simple trigonometry and some paper engineering, and came up with the device which can be seen in operation in Figure 1. The protractor is reading an angle of 52.1° and it can be seen that the scale, running from 49° to 58°, is expanded approximately 10x compared to a circular arc of similar size. This means that tenths of a degree can easily be shown. The disadvantage of the scheme is that the scale is non-linear.

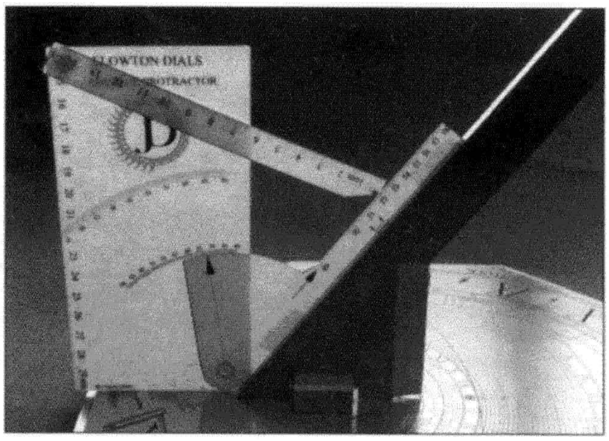


Fig.1. A Pocket Gnomon Protractor

Figure 2 shows the three major pieces of the protractor. If the reproduction of the Bulletin is accurate enough, this can be photocopied to allow readers to make their own device. It does not matter if the reproduction or photocopying gives a slight change in scale, as long as the magnification is the same in the horizontal and vertical directions. The paper needs to be laminated in a plastic pouch to give it protection and stiffness. The thicker (250 microns) of the two standard pouches is preferable. After laminating, the three pieces are cut out carefully with a scalpel and straightedge.

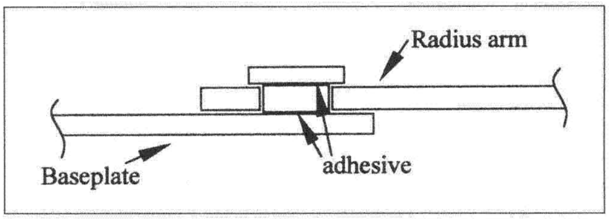
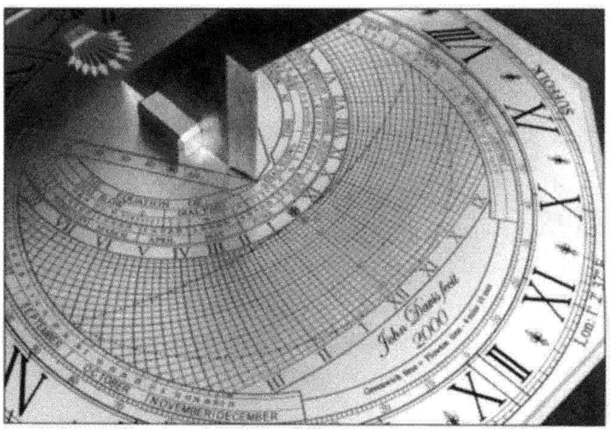
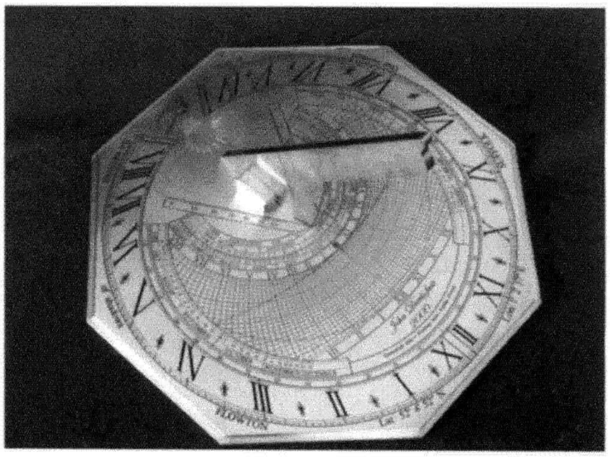


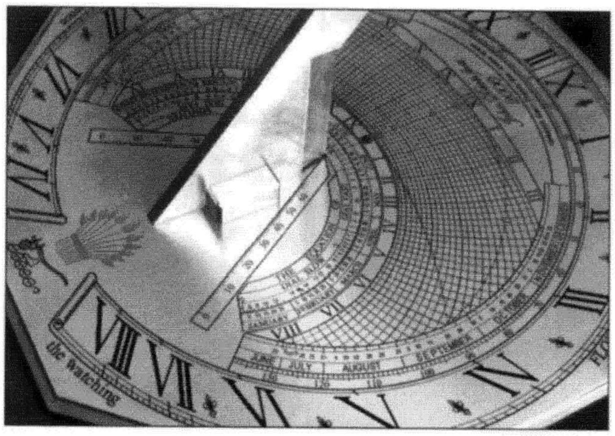
Fig.3. Schematic Cross-section of pivot assembly

The lines which are critical for the accuracy of the protractor are indicated and printed with narrow lines. The accuracy also depends on a smooth hinge action, with the centres properly located. This is achieved, whilst keeping the overall thickness down to 0.75mm, by the scheme shown in Figure 3. Disks of 6mm diameter are cut from the two swinging arms by using a punch made from a piece of sharpened tube. The cut disks are carefully glued with contact adhesive onto the base piece on the appropriate centres. The holes in the swing arms are pushed into position over the disks and the arms retained by two more disks, 8mm in diameter and cut from a scrap area of the laminate.





The protractor has three auxiliary scales. The small circular one simply repeats the main linear one, but has an increased range of 40-90°. The larger circular scale runs in the opposite direction and is for those occasions when the protractor must be stood alongside the gnomon. It has a range from 45° to 90°. Note that is read against the fiducial edge of the radius arm, as shown by the pair of arrows around the 51° mark. Both these circular scales can be read to 0.5° if your eyesight is good enough. The third scale is simply a folded ruler for measuring dial plates. Remember to check the magnification of the photocopying before relying on it!



In practice, the main body of the protractor is stood on the dialplate and the radius arm is swung down so that it rests on the gnomon. These two pieces are held in position between the thumb and forefinger and then the indicating arm is swung down until it just rests on the radius arm. The gnomon angle can be read off directly at the touching point.

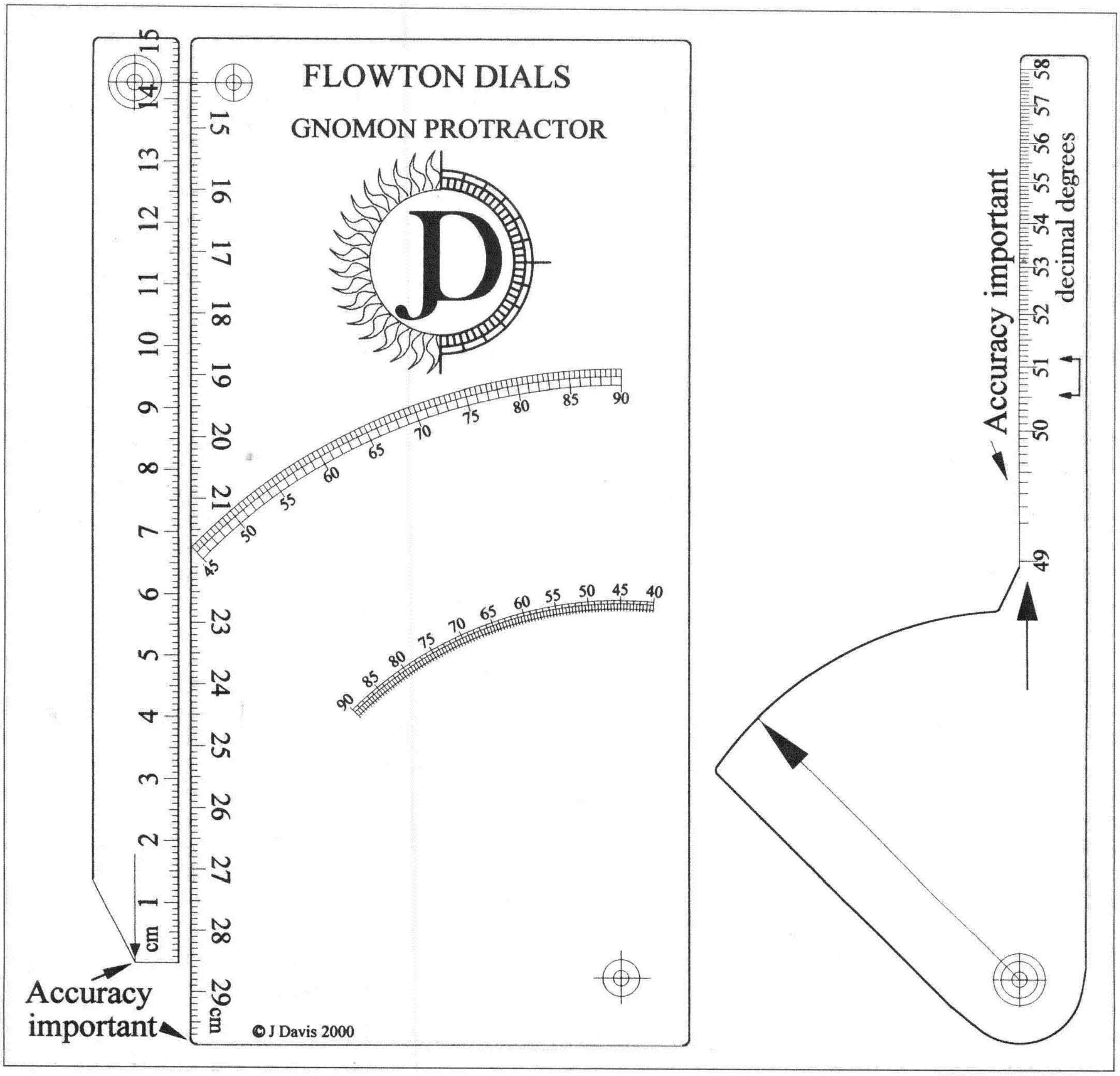


Fig.2. Real-size drawing of the main components of the protractor

The geometry of this protractor is not unique and it could easily be redesigned in different sizes or for different minimum angles. A spreadsheet is available from the author for readers who would like to design their own. It would be possible to linearize the scale by placing the fiducial edge around an appropriate curve. I have no means of cutting the required shape with sufficient accuracy, so dispensed with this idea. Although the device in no way competes with proper engineering vernier protractors, it is convenient for measuring dials which are found unexpectedly, and is cheap enough not to matter if it gets damaged or lost.

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A BRIEF HISTORY OF THE BRITISH SUNDIAL SOCIETY

DAVID YOUNG

PART 1

INTRODUCTION

The interest in a Society's early history tends to grow as it passes its 25th anniversary and at this time or when the half-century is reached someone decides to do some research and produce a history of the association. Unfortunately, as happened with my own Historical Society, many of the originators had died, leaving only minute books (not such a good source of factual material as one might suppose) and a few second hand personal memories. It is fitting therefore that we, at the tender age of about ten years, should attempt to put on record the foundation and early years of the society

Having been concerned with the society from the beginning, I have been asked by our Editor to write such an account and I am very pleased to do so. It is to some extent a personal story and as I have been so close to these events they cannot be completely unbiased, although I am fortunate enough to have preserved practically all of the correspondence between the founders in those early years and most of the documents and letters sent and received during the ten years I have been Secretary. All these records will be placed in the society's archives to help future historians to make their own independent account.

THE BEGINNING OF A DREAM

During the later half of the last century there appears to have been a resurgence of interest in sundials and there is every possibility that the thoughts of starting some kind of Association had occurred to many. Certainly Christopher Daniel and Gordon Taylor had talked of the possibility in the early seventies. Both were eminently qualified to do so but unfortunately being in full time employment they understandably did not have the time to follow up their ideas.

My own interest in Sundials happened by chance having picked up the book,"Sundials Old and New" by A P Herbert in our local library. This book has often been criticised but it has to be said that it has been instrumental in awakening an interest in the subject for many of us. After reading other books by Waugh and Rene Rohr and spending a long holiday photographing dials across the country I felt conceited enough to add 'The Time of the Sundial' to my repertoire of talks that for some years I had been giving to local History Societies. Luckily enough so little is known about the subject by the general public that I soon became the local expert ....in the country of the blind the one eyed man is king!

During this time a colleague of mine pointed out to me an article on sundials by a Dr Andrew Somerville published in the Scots Magazine (June 1986). It was about the research he had been doing on the polyhedral dials in Scotland and significantly at the end of the article he gave his address asking those interested to contact him. This resulted in some correspondence between us. I asked him if there was such a thing as a sundial society and in his reply (dated 25th July) he mentioned that there were active groups in Holland and Germany but none in this country. He had mooted the idea of forming a section of the AHS (Antiquarian Horological Society) but added that it needed a driving force to get it going and at the moment he had no time to do so. He went on to say "I had hoped to twist the arms of some of the professionals in the field, especially Christopher Daniel of the National Maritime Museum at Greenwich, but he has remained resolutely untwisted!"

After this exchange my wife and I visited Andrew's home in Cheshire where we were royally entertained by him and his wife, Anne. The most significant thing about that visit was being able to talk to someone else about the subject. One can learn a lot from books but one can also pick up wrong ideas; and I even found out how to pronounce analemma!

In November of that year when I gave a talk to a society in a neighbouring town a lady from the audience afterwards told me that she was acquainted with Christopher Daniel who was the author of a recently published Shire book on sundials. She said that he had been surprised to hear that there was someone in Essex giving talks about sundials, and that he would probably like to hear from me. This gave me an ideal opportunity to write and explain what I was doing, with the limited experience I had. In my letter of 16th November 1986 I said that a National Society would be the the ideal to aim for: "I envisage a society open to Amateurs and Professionals alike, with the overall responsibility for cataloguing existing dials in the UK, promoting the exchange of information and encouraging the preservation and restoration of our older dials".

This resulted in a prompt reply and an invitation to meet him at the National Maritime Museum where until very recently he had been Curator of the extensive Sundial Collection. The meeting which took place in January 1987 again proved to me the advantage of personal contact (quite apart from being treated to a free lunch) and he was enthusiastic about the formation of a society but, like Andrew, felt that due to pressure of work he could be of little help at that time.

So the matter rested for almost two years with just the occasional telephone call between us.

A NEW INITIATIVE

Quite out of the blue in November 1988, I received a telephone call from a Mr Charles Aked saying that he had heard from Andrew Somerville that I was willing to help in the formation of a sundial group and would I be interested. He said that the three of us would be enough to form a provisional committee and possibly Christopher Daniel might be willing to join us. I indicated that I thought he would, but that before giving a final answer to his invitation I would like to consult with Andrew Somerville. Within a few days a letter from Charles Aked dated 8th December expounded further his thoughts on the subject. He had suggested that the group could be affiliated to the AHS which would probably give us some sponsorship. It would be a useful start but at the same time it might restrict us and there were some advantages in being an independent body - if so he would ask Dr Ward, formerly of the Science Museum, to become its President. He had also thought of a name for a quarterly newsletter, "The Dial". As things seemed to be progressing fast I made arrangements to visit the Somevilles, and did so on 7th January 89. Andrew had, at this stage, time to spend on the project, so after contacting Christopher Daniel he arranged a telephone conference between us where we agreed to go ahead provided that we could find enough potential members. Andrew drew up a suitable letter to send to all those known to have some interest in the subject and to a number of specialist magazines. A spate of letters between us arose within the next few weeks. Charles Aked discoursed at some length on the possible name for our society counselling us to beware of titles that might have unfortunate acronyms such as the "Society of Dialists" He also suggested that a title with the word "gnomonics" might be thought by the public to be a society for the preservation of garden gnomes! More seriously he backed my earlier suggestion of "The Sundial Society of Great Britain" and his first attempt at a constitution had that title. He also readily agreed to edit a society journal. I laid out plans for a fixed dial recording scheme, I planned the basis for a scheme to record fixed dials using pads of recording forms; and I would be responsible for the membership side of things, and act as treasurer for the time being. In this latter capacity I estimated that at a membership fee of £5/£10 per year we would need a minimum of 50 members to be able to produce a quarterly duplicated bulletin. At this stage Andrew was definitely the driving force but we were all full of enthusiasm and were buoyed up on each occasion when we received a phone call - "We have two more from the Newcomen Society" or "Another from Astronomy Now".

The numbers slowly increased and as they did so we felt more confident that our dream would become a reality. Although we kept our options open we felt fairly sure that we would want to be an independent society. However in a letter dated 23rd January to Mr Tom Robinson of the AHS, Andrew complains about being beleaguered by AHS officials who would like the group to be a section of their society. He goes on to say in defence of our decision for independence " I was concerned to hear from Mr Penney (*the journal editor*) that he had turned down an article on a modern high precision sundial on the grounds that it is an Antiquarian journal and hence not the appropriate place for it" In the event the majority of replies to our exploratory letter were not from AHS members so the decision was firmly made by all four of us to opt for independence.

By April 1989 we had over 30 committed to membership and Charles Aked had submitted some draft rules for the society which were subsequently shortened somewhat.

Andrew commenting in a letter of thanks to Charles says "They seem very comprehensive, though I must confess I am wary of having an over elaborate constitution, having suffered in the past from 'barrack-room lawyers' more interested in the interpretation of the rules than in the real business of the society"

Christopher Daniel had for the past year been the author of a sundial page in 'Clocks' magazine and he devoted the May issue to an account of the formation of the society. Realising that when this was published it would bring in more members, Andrew decided that we should now start to organise the next phase of the operation. In this respect he had drafted a letter to be sent out with membership applications and suggested that we have a formal meeting soon. I invited him to stay at our house at Chingford and it was agreed to hold our first official meeting there on Friday 5th May 1989. This would in fact be the first time we had all met together and it would be the inauguration of the British Sundial Society

...to be continued.

112 Whitehall Road,
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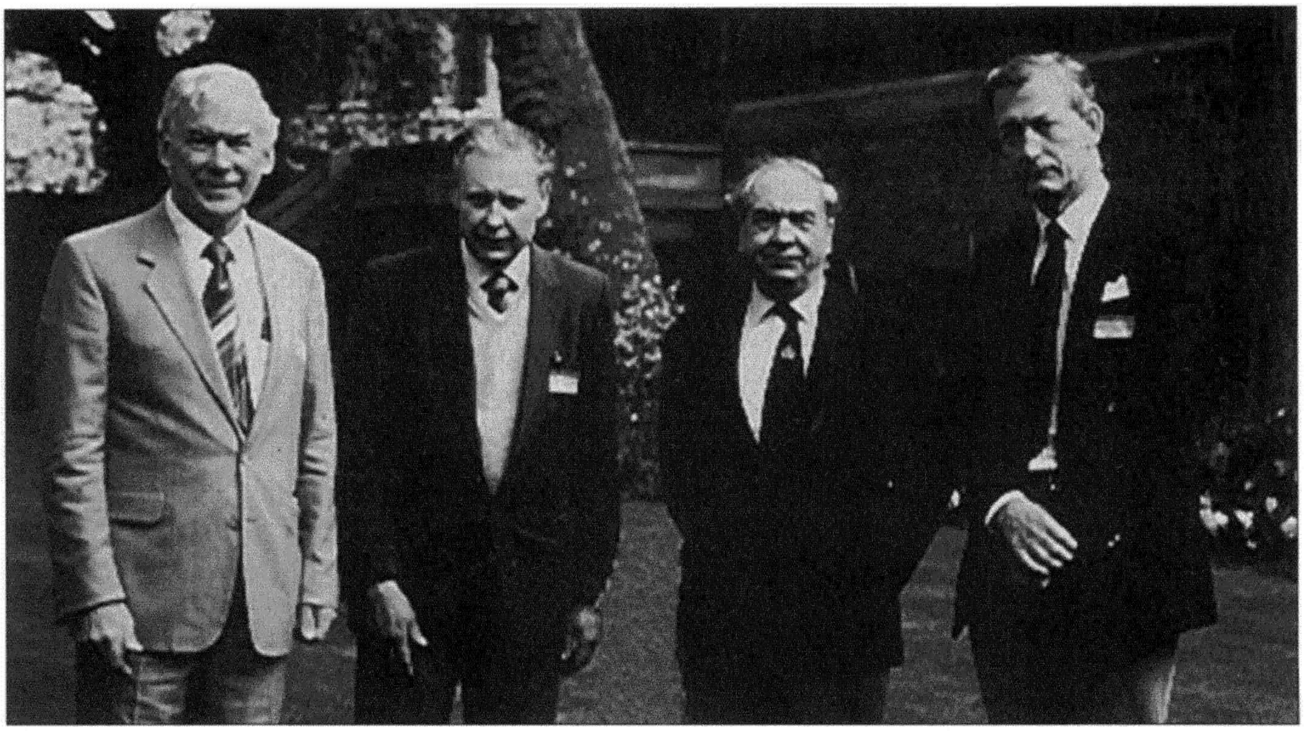
London, E4 6DL



The Founders: Andrew Somerville, David Young, Charles Aked, Christopher Daniel

RICHARD TOWNELEY AND THE EQUATION OF NATURAL DAYS

TONY KITTO

The last quarter of the 17th century was an important period in the history of sundials. By the end of this period pendulum clocks and balance spring watches were readily available, but needed calibration using a sundial and an Equation of Time table. This article examines Flamsteed's early work on the Equation of Time. It highlights some of the activities of other people who contributed to that work; and it is based upon detailed letters which Flamsteed sent to his friend Richard Towneley in the years 1673-1688.

RICHARD TOWNELEY THE VIRTUOSO

Richard Towneley (1629-1707) was interested generally in mathematics and natural philosophy and in his day was called a virtuoso, someone skilled in curiosities. He was the head of a wealthy Catholic family in Lancashire. The Towneley family took great care to provide their children with a good education in continental Europe. The library at Towneley Hall, their home, was well stocked with a wide range of books including all the major books on astronomy and mathematics[1].

Richard Towneley was particularly interested in astronomy, clock-making and the weather. He was the first person in England to record and publish regular rainfall measurements[2]. He was also an early user of barometers for weather forecasting. When Robert Boyle published an account of the relationship between the pressure and volume of air in 1662, he described what was later to be known as Boyle's Law as 'Mr Towneley's hypothesis'.

Towneley published little of his own work; but in 1667 he sent a letter to the Royal Society[4] which had far reaching consequences. Adrien Auzout had claimed a French first in inventing the micrometer. Towneley wrote to point out that Auzout was not the first person to have developed such a device. The English astronomer William Gascoigne had developed a micrometer before the Civil War. Towneley had produced his own version of that micrometer and was using it in Lancashire. The Royal Society showed great interest in Towneley's micrometer and he sent one to the Society.[5] This micrometer afterwards went into the hands of Sir Jonas Moore, the King's Surveyor~General of the Ordnance, a good friend of Richard Towneley, who made the best possible use of the instrument by giving it to John Flamsteed.

THE FLAMSTEED-TOWNELEY PAPERS

In 1707, John Flamsteed (1646-1719) wrote an account of his early life. This was published in Francis Bailey's *Account of the Revd John Flamsteed* (London 1835) and for many years it was the only readily available information on Flamsteed's early career. It tells us that Flainsteed first visited London in 1670 and was given the Towneley micrometer. He visited Towneley Hall and was given advice on how to make best use of the instrument. In 1891, seventy letters came to light in the library of Moor Hall, Harlow, Essex. These letters, written by Flamsteed to Richard Towneley over a period of fifteen years from 1673 are now in the library of the Royal Society IMS 243J and provide a unique insight into the early work of Greenwich Observatory. In recent years Flamsteed's surviving correspondence has been published in ful l.[6]

In March 1675, after Flamsteed had been appointed the King's "astronomical observator", his letters to Towneley arrived regularly. The main topic of correspondence was not his job or the plans for the new observatory. It was about a new spring watch made by Christiaan Huygens. Robert Hooke claimed that he had made the same invention some years earlier. Thomas Tompion was involved in Hooke's plans to market a spring watch and this held up plans for Tompion to make Flamsteed a pendulum clock for his experiments.[7-9] Then in September 1675 Flamsteed sent Towneley:

"a notion wholly, for ought I know, new to put into experiment, briefly the equations of Natarall dayes are yet in Controversy amongst us, and though I have demonstrated in the Diatriba printed with Mr. Horrox remaines, that the Astronomica can be only true, yet it is question whether the dayly retume of any meridian on our earth to a fixed star be equall and Isoclironical at all times of the yeare".[10]

Re-phrased in modem language, Flamsteed wanted Towneley to help him to prove that the Equation of Time was valid and that the earth rotated at a constant speed throughout the year. Before discussing the reasons for the letter and what happened subsequently it is useful to provide some background information on the understanding of the Equation of Time at that date.

EARLY PUBLICATIONS OF THE EQUATION OF TIME

J. L. E. Dreyer, writing in the Encyclopaedia Britannica in the 19th century, claimed that Flamsteed was "the first who explained the true principles of the Equation ofTime"[11]. In fact there were a number of related publications prior to those of Flamsteed. All the ingredients for calculating an equation-of-time table were available to astronomers in Thomas Streete's *Astronomia Carolina* published in 1661[12]. Christiaan Huygens created the first accurate table based upon the Equation of Time for the book *Kort Onderwys*, published at The Hague in 1665. It was translated into English and published in Philosophical Transactions in 1669. Huygens simply explained how to calibrate a clock using a table of the differences between an accurate clock and the sun at noon for every day of the year. He did not explain the mechanism, beyond the phrase: "Here take notice, that the earth makes an entire revolution in the Ecliptick in 365 days, 5 hours, 49 minutes or thereabout and that those days reckoned from noon to noon, are of different lengths; as is known to all, that are versed in Astronomy"[13]

John Wallis was probably the first person in England to publish an explanation when he wrote "*his Hypothesis about the Flux and Reflux of the Sea*" in 1666. He said there were at least two causes for the inequality of the natural day. The first cause was the variation in the Earth's annual orbit around the sun, given by the tables of the sun's annual motion. The second cause was the tilt of its axis to the plane of the Earth's orbit, given by the table of the sun's right ascension. Wallis said that not everyone agreed whether there were any additional causes.[14]

In January 1670, Flamsteed reported to John Collins, a member of the Royal Society, that he had written an equation of time around 1667[15]. The word 'equation' did not then have the mathematical meaning as we know it today. It was simply a set of numbers to add to an initial value to improve the expected results. The starting point was the mean position of the sun each day to which was added an equation called its equation of centre. It was then straightforward arithmetic to forecast the difference between the mean and apparent time of noon each day. Flamsteed did not own a pendulum clock and had no means of checking his forecasts on a daily basis. For Flamsteed, the equation was just the first step in a series of equations that he hoped would eventually forecast the accurate timing of solar eclipses and the position of the moon and stars.

Flamsteed eventually sent Collins his equation table with a detailed explanation in 1671.[16] Wallis included it as an appendix to the papers of Jeremiah Horrox which Wallis was editing. Horrox was a brilliant astronomer from Lancashire who had died young in 1640. His work had a significant influence on Flamsteed. The papers were published in 1673 [17] and were spoken of as '*Mr. Horrox remaines*' in Flamsteed's letters to Towneley in 1675.

There were other equation tables published around this time such as that by the Frenchman Gabriel de Mouton in 1670. The equations of Flamsteed, Huygens and de Mouton were equivalent but each expressed in a different form. A letter from Flamsteed to Collins in 1672 explained his approach. After agreeing with the values in de Mouton's equation tables, Flamsteed wrote that "what he alledgeth that this aequation is to be applied to the usuall Aeras. I find not necessary, for equall time is not Aerall, but caelestiall". Flamsteed understood that the Equation of Time must be fixed not to some man-made calendar date but to a particular astronomical event. That event was a time when "the Earth on its Aphelion or Perihelion was in the first point of some Cardinalle signe, where there was no aequation of time, but the same moment was both the Caelestial, and apparent time".

Flamsteed realised that the position of the earth's perihelion changed slowly each year. At some time in the past the perihelion had coincided with the winter solstice and at that moment the Equation of Time was zero. He had calculated his equation tables on that basis. Huygens had chosen to select February 10 as the zero starting point for his equation table such that the time would reach its greatest value on November 1st before reverting to zero in the following year. It was adequate for calibrating clocks but was not useful for long range astronomical forecasts. Flamsteed pointed out to Collins that his own way of determining the value of the Equation of Time was "the only one for conveying them uncorrupt to posterity". In this respect, Dreyer' Encyclopaedia Britannica article appears to be correct in its claim.

A copy of Flamsteed's table is included here. The table gives his equation of natural days in terms of the sun's position in the zodiac. From earth, the sun appears to move against the stars through 360 degrees in the period of one year. Astronomers, from time immemorial, had divided the sun's movement into twelve equal parts, called Signs, each of which consequently contains 30 degrees. The names and symbols by which they are characterised are as follows: -

North of the Equator		South of the Equator	
Aries	♈	Libra	♎
Tauru	♉	Scorpio	♏
Gemim	♊	Sagittarius	♐
Cancer	♋	Capricomus	♑
Leo	♌	Aquarius	♒
Virgo	♍	Pisces	♓

Flamsteed's table begins with the place of the sun at the spring equinox, known as the first point of Aries. The angular distance of the sun from this point is called its *Longitude*. If you take the sun's Longitude from an almanac when measuring apparent time, the table gives the equation of time to be added (A) or subtracted (S) from the apparent time in order to give the mean time.

THE CONTROVERSY

A key phrase in Flamsteed's earlier letter to Towneley was "the Equation of Naturall dayes are yet in Controversy amongst us". Earlier in 1675, Juan Cruzado, Chief Pilot of Spain, had written to Henry Oldenburg primarily concerning a proposal for the world's prime meridian. Oldenburg passed the letter on for replay to Flamsteed in his new position as Astronomer Royal. Flamsteed showed little interest in the question of which place in the world was chosen for the prime meridian so long as other places were accurately defined from it. What did concern Flamsteed though was another point made by Cruzado, who claimed that hit clocks did not show any differences in the length of days. Flamsteed believed that Cruzado's clocks were not very accurate and probably lost too much time when they were wound every day.[19] Flamsteed still had no clocks of his own so he was not in a position to contradict Cruzado. This seems to be the reason that Flamsteed requested Towneley's help.

Flainsteed wanted Towneley to compare his clock measurements for 14 days around the autumn equinox and the following winter solstice as he expected to see a difference between the two periods of around 50 seconds a day.

In the next letter Flarnsteed also asked Towneley to measure the time a star would take to return to the same azimuth both at the aphelion and perihelion.[20] Flamsteed wished to find whether the speed of the earth's rotation perhaps varied with its distance from the sun. This possibility was used by Kepler in 1599 to explain why the movement of the moon was so difficult to forecast.[21]

Unfortunately none of Towneley's original letters to Flamsteed survive but luckily a copy of one, dated November 24th 1675, still exists. Towneley agreed to take measurements around the solstice and claimed "I find I can tell when the sun comes to the meridian to a second' or two at most". He then went on "Mr Hugens in his book of pendulums saith , that his exactly followed his Equation, and I have heard Sir J. Moor say, that My L. Brunncker had a clock, which went within 3. minuts in a year: By his means you may learn, how he found the Equation"[22]. The 'book of pendulums' mentioned by Towneley was Christiaan Huygens: *Horologium oscillatorium* (Paris, 1673).

It is clear that a number of other people had already made the sort of measurements suggested by Flamsteed. The diary of Robert Hooke included the following entries:

Sunday 13th December 1674 Gave Tompion a description of Aequating of time for Sir J Moore's Clock

Sunday 3rd January 1674/5 At Sir J. Mores Found his clock faster thin the sun between 3 and 4 minutes.[23]

A letter to Towneley in December 1675 tells us that Flamsteed now had his own clock and was beginning to find how unreliable clocks still were at this time, with dust mixed with oil clogging the wheels. It also shows that Flamsteed had now read *Horologium oscillatorium*.[24] A letter from Flamsteed to Sir Jonas Moore, dated 20 December 1675, provides further evidence that Flamsteed realised he needed two good clocks if he was to make a serious attempt to validate his equation tables and that Moore would be the man to pay for them.

REPORTING THE RESULTS

In January 1676, Flamsteed sent a letter to Oldenburg to pass on to Cruzado. It included the results of Towneley's experiments to show that the days around November were longer than the ones around September. Flamsteed was able to confirm them with his own experiments and enclosed a copy of his equation table saying "Even though constructed for the year 1672, it can be employed through the whole course of this century without significant error".[28] Flamsteed, with an eye on his own requirements, exhorted Cruzado to equip himself with instruments of greater precision. Moore had obviously already got the message and in the next letter to Towneley Flamsteed was happy to write that Tompion would be providing the Observatory with two new clocks.[27]

The clocks arrived in July 1676 and the experiments proved troublesome as many subsequent letters to Towneley testify. In March 1678 Flamsteed wrote to Moore that "the Isocroncity of the earths revolutions was onely supposed not demonstrated by me but your clocks have proved that rational conjecture a very truth".[28] However Flamsteed never published his results and instead he concentrated on recording accurate positions of the sun, moon and stars. In April 1679, Flamsteed told Towneley that he had made new solar tables from his observations of angular distances of the planet Venus from the sun and the stars. These gave him a better estimate for the aphelion and an improved solar equation.[29] He now updated the Equation of Time and he continued to improve it for the next twenty years.

He seems to have mentioned the clock experiments only once after Moore's death in August 1679. This was in a letter to Bishop Seth Ward at a time when Flamsteed was in danger of losing his allowance in Charles II's retrenchments. In listing his achievements to date at the Observatory he wrote "The Aequation of time I can prove by many and careful experiments to be no other thin Astronomical."[30] We can take this to mean that Flamsteed had been unable to find any changes in the speed of the earth's rotation at any time of the year.

POSTSCRIPT

In 1686. Isaac Newton provided Flamsteed with information from a forthcoming book that is now known as the *Principia Mathematica*. Flamsteed passed the news to Towneley with the words "Mr. Newtons Treatise of Motion is in the presse... I congratulate my owne happinesse".[31] He knew that there was confirmation that the earth rotated at a relatively constant speed.

Towneley Hall is now an Art Gallery and Museum. It is hoped that in 2001 a sundial will be installed at Towneley to commemorate Richard Towneley and his small contribution to the study of the equation of natural days. The sundial is being designed by a member of the British Sundial Society, Alan Smith, and it is hoped that full details of the sundial will appear in a future edition of the Society's Bulletin. In the meantime more information about Towneley Hall and the progress with the sundial can be found on the following web site: www.burnley.gov.uk/towneley.

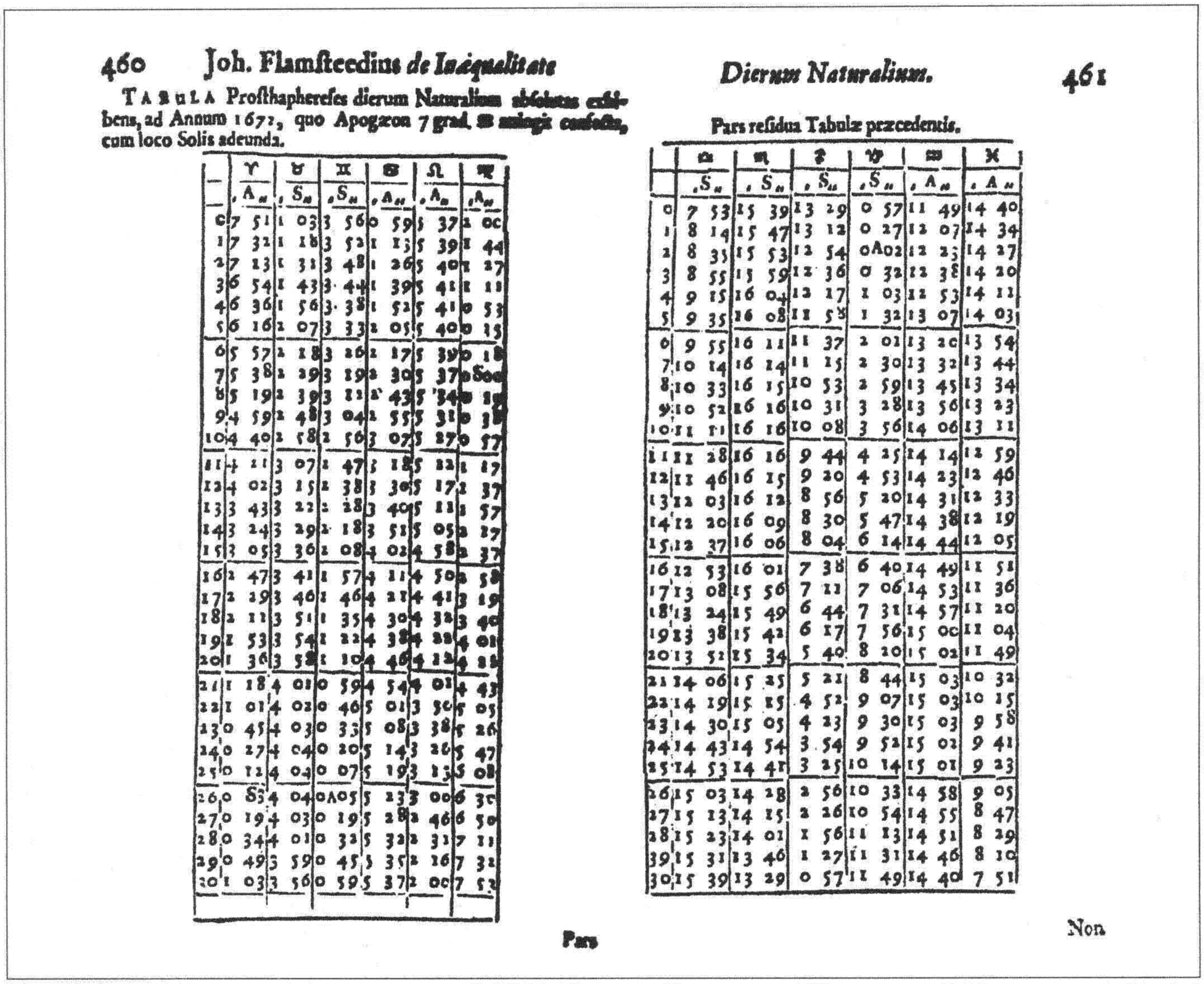


Fig.1. John Flamsteed's equation of natural days for 1672

ACKNOWLEDGEMENTS

I thank Douglas Bateman for suggesting that I submit this article, a version of which was previously published in Antiquarian Horology in December 1999.1 also thank Alan Smith for advice and information on the proposed sundial.

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UTTOXETER MILLENNIUM MONUMENT

ANDREW McVEAN

About three years ago Uttoxeter Town Council was determined to mark the beginning of the new Millennium in some tangible way; at that time their thoughts centred around the erection of a Victorian bandstand. When Stephen Smith, a new member of the town's Rotary Club, voiced his opinion that this was not a very exciting way of doing so, he was challenged to think of something better. He did just that.

His concept was that something special was required to provide a long lasting memorial to the birth of the new millennium. Its theme should be the solar system because this is the new frontier that challenges us in the new millennium, in the same way that the exploration of the earth challenged mankind in the last millennium. It should inspire thought about the exploration of the solar system and fire the imagination. It could also incorporate a sundial as an active link between the sun, our earth and the rest of the solar system. It would tell the time to an onlooker in a fundamental way that has not changed in the past and will not change in concept as long as the solar system exists. Lastly, whatever was to be made should celebrate Uttoxeter, and should be made by local firms and their craftsmen.

Some three years later - after much work in design, obtaining funding via a mixture of sponsorship and council funds, obtaining planning permission, manufacture and the excitement of installation - Stephen Smith's design has been realised. Uttoxeter's Millennium Monument is sited for all to see in the town's market place, beside the memorial erected to Samuel Johnson, who was closely associated with the town.

Fig.1. The monument in the Market place, beside the Johnson Memorial.

The monument consists of a large stone disc supported on a stone base. The diameter across the top is 2000mm, symbolising the 2000 years that have passed since the birth of Christ. Four bronze quadrants are let into the top, forming a slight dome. The top surfaces of the quadrants are of a rough texture to represent Space. The abutting edges of the quadrants are bevelled and polished, and set to lie North/South and East /West. Stainless steel letters - N, E, S, W - are set into the stone at their extremities so that the whole domed surface looks like a correctly oriented compass card.

Granite hemispheres are set into this dome to represent the planets, and the angular displacements between them are the same as the angular displacements of the planets at midnight on 31st December 1999. Different granites are used for each model planet so that their colours are similar to the colours of the planets as seen from earth. Their diameters are to a logarithmic scale to cope with the vast differences in the diameters of the planets. Similarly, their distances from the centre are also to scale - a different logarithmic scale to accommodate the even greater distances involved. A brass hemisphere 125mm in diameter (to the same logarithmic scale as the diameters of the planets) is set into the centre and represents the sun.

A simple armillary sundial consisting of a support ring, equatorial ring and gnomon is mounted above the sun. An early design had a horizontal dial, but an armillary dial was subsequently chosen as it does not have a sharp gnomon to hazard revellers pouring out from the nearby pubs!

Fig.2. A view from above

A stainless steel handrail surrounds the monument, supported on brackets from the upper stone disc. Concealed within the monument is a time capsule, within which the best efforts of Uttoxeter's schoolchildren have been placed, together with some keepsakes from the town's main industries.

Four bronze plaques are mounted on the bevelled surface of the stone disc, and set at the cardinal points. Each is dedicated to one of four main industrial concerns in and around Uttoxeter, and gives a brief description of the history of the company. These are JCB, Uttoxeter Racecourse, Elkes Biscuits, and Barrett Industries. These companies generously provided the major part of the funding for the monument.

A further twelve plaques have been formed by sandblasting their outlines and text into the stone, and colouring the indentations made by this process. They are set equally between the four bronze plaques. Between them they detail the makers of the monument including the main contractors Hayes Industries, summarise the history of Uttoxeter, list Uttoxeter's achievers, schools and organisations, provide a memorial to Bartley Gorman (Uttoxeter's Gypsy Bare Knuckle Champion 1972-1992), give a message from the churches in the town and another message from the Mayor, provide some geophysical data about the position of the monument, list the remaining sponsors of the monument, and celebrate its opening. One plaque has been left blank to allow the recording in the future of significant Uttoxeter events or personalities.

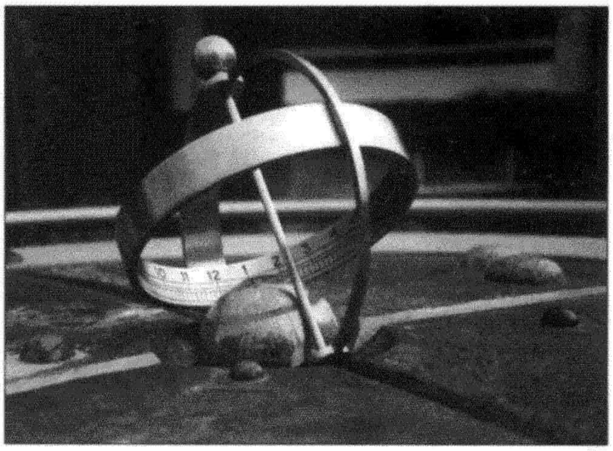

Fig.3. Detail of the Armillary Dial

The monument was unveiled on 23 September 2000 by The Earl of Shrewsbury and Talbot. Few people pass by without stopping to look at it. All associated with the project are proud of it, and are confident that it will still be in place as a monument to the birth of the third millennium long after the Dome at Greenwich has passed into history.

ACKNOWLEDGEMENTS

I am grateful to Stephen Smith, the designer of the monument, and to David Whitmore, the Chairman of Uttoxeter Council's Millennium Monument Coordinating Committee, for their help in writing this article.

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A REMOTE READING SUNDIAL USING FIBRE OPTICS

MIKE SHAW

I wanted to make a sundial which overcame some of the perceived difficulties associated with dials, i.e.:-

a) The need to go outside to read the time.
b) The confusion caused by a 24 hour display, (we are all used to 2 x 12 hours in a day).
c) To be usable when the sun isn't shining (you will find that I cheated with this one)

I decided that the answer was to make a sundial using a two component system, with a collector (in the garden) linked to a remote display unit (in the house), utilising fibre optics.

Fibre optic cables (FOCs) have been used to make sundials before of course [1,2] I make no claim of originality in that respect - though this is the first of this particular design (as far as I know)

The collector (Fig 1) is half a plastic pipe cut lengthways. I used 160mm length by 150mm diameter. The main problem was to find a pipe that did not change shape when cut down the middle. I eventually used a pipe used for underground drainage (5mm thickness) which was stable, but I had to try quite a few different types before finding one which was suitable. The ratio of length to diameter was not critical, so long as it would cope with the maximum variation in sun's declination over the year; I allowed plenty of margin.

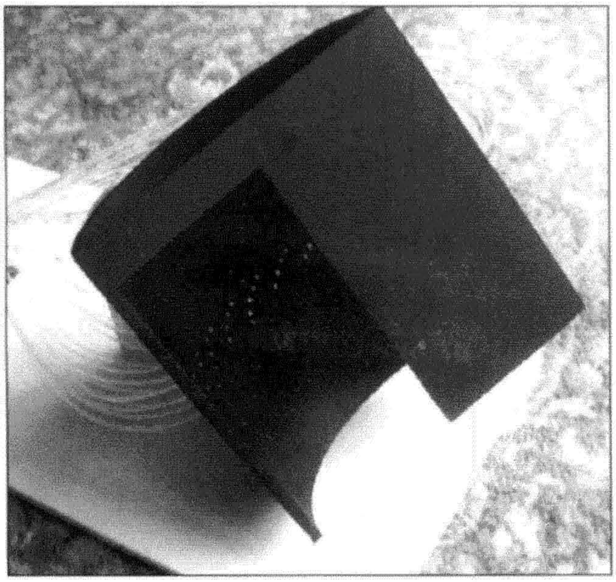

Fig.1.

I also had quite some difficulty getting suitably "low tech" (low cost) FOC. It was easy to find high priced sheathed FOC as used in communications. I eventually managed to get some 1.5mm diameter unsheathed FOC from a theatrical lighting supplier - they use them for making "starry night" backcloths for stage productions. They sold me one of their used "looms" for a nominal charge.

At 1.5mm FOC, the 150mm diam collector gave enough "space" to have one FOC every 5 minutes without overlap, but I opted for one every 15 minutes for this, my first attempt. I marked out the spacing on paper using vertical lines, then attached the paper to the inside of the half cylinder. The 1.5mm holes were drilled offset vertically to keep them distanced. In Fig 1 you can see that the holes were drilled in groups of 4. This made it easier to identify them later at the assembly stage.

Originally I was considering using a cylindrical lens to focus the sunlight[1], but trials showed this to be an unnecessary complication. In practice, stray light was more of a problem. Hence, the collector was sprayed matt black to keep spurious reflections to a minimum.

My first plan was to cover the face of the collector, leaving a narrow slit down the centre, in order to light up one cable end at a time, rather like Robert Adzema's "Crack of Dawn" dial.[3] However, this created a problem, as the width of the light ray varied depending on the time of day. You could, therefore, get times in the early morning and late evening when the ray fell between FOCs and none of them was lit, or get more than one being illuminated around noon. I

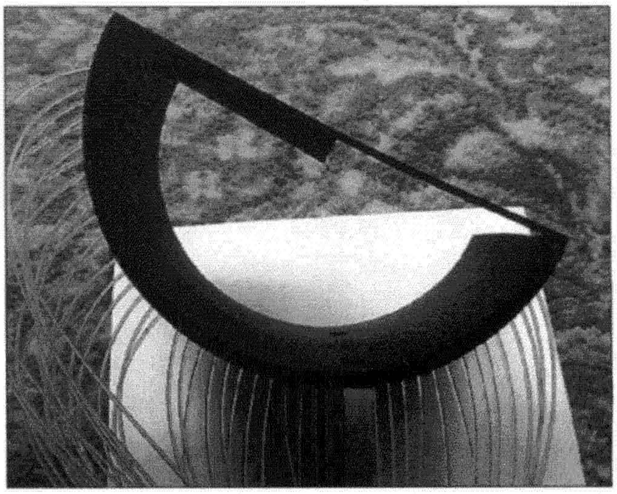

Fig.2.

therefore decided to cover over half the collector as shown (the upper crosspiece is there to give a degree of rigidity and ease of construction) The cover was made from thin, rigid "Formica" type laminate. Thus, as the sun progresses, more and more of the FOCs are illuminated.

The half cylinder collector was mounted equatorially (53 degrees here), and was offset from noon to collect the sun during the 12 hour period 9am to 9pm - see Fig 2. (I no longer have to get up at 6am, so I don't). It was set for daylight saving time, on the basis that that's when the sun is most likely to shine, and also that, in the UK, we are on daylight saving for 7 months out of the 12. I also included a 3 degree longitude west offset.

Thus, in use, with the collector positioned pointing due south, at 9:15am, the first FOC is illuminated, at 9:30 FOCs 1 and 2 are lit and so on. At the display end, an increasing number of FOCs are sequentially illuminated, with the last one clockwise indicating local apparent time to the nearest quarter of an hour. Early FOCs eventually go into the shade of the cylindrical collector as the sun progresses. Towards the end of the day, less and less are illuminated.

A bracket was mounted on the baseboard to represent the hole in the side of the house through which the FOCs would be threaded- see Fig 3. In this prototype, collector and display are a mere 450mm apart. In use, the collector would be in the garden, and the display inside the home.

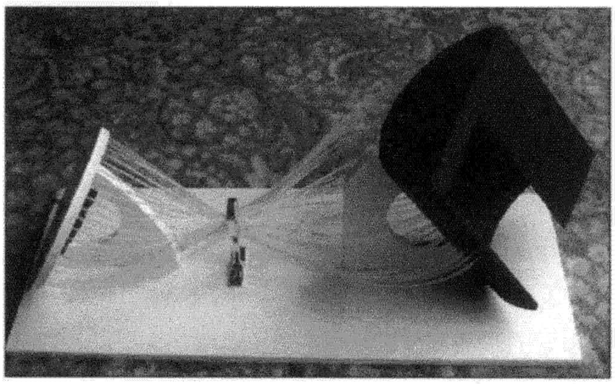

Fig.3.

The display (Fig 4) was made using a painted 150mm diameter aluminium clock face, mounted on 8mm plastic board. Some thickness was required to hold the ends of the FOCs securely. The FOCs are a tight sliding fit at each end and are not glued in place. The picture shows the illuminated FOCs at just before 3:15pm - in reality, there is no doubt about the time shown by the FOCs if the sun is shining - the photo doesn't do justice to the difference between illuminated and shaded FOC ends. It is worthwhile to cut the display ends of the FOCs at an acute angle. This makes the display more readily visible at an angle.

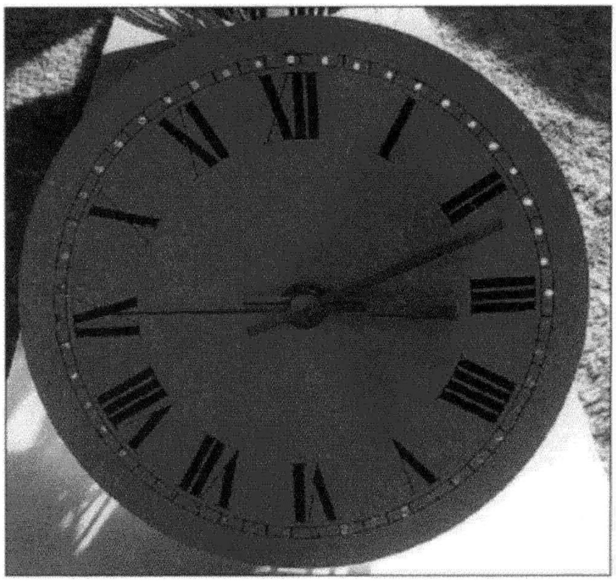

Fig.4.

And finally, just in case the sun doesn't happen to be shining, I added a clock movement - you can therefore get the difference between clock and local apparent time. This is the cheating bit!

Several improvements could be incorporated

- a rotatable collector, thereby adjustable for daylight saving and equation of time changes
- an increased number of FOCs
- interchangeable collector covers with the edge shaped to adjust for the equation of time (two would be needed)

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JOURNAL REVIEW

COMPENDIUM, JOURNAL OF NASS:VOL.7, NO.3 (SEPT 2000)

This issue of the North American Sundial Society's Journal opens with a description of the beginnings of a 'Sunwheel', a mini-Stonehenge being built in a field on the campus of Amherst, Massachusetts. Its designer is a Professor of Astronomy at Amherst, and the Sunwheel is to be an educational resource for schools, for university students (UMASS) and for members of the public. A start was made on the 'Wheel', approximately 30m in diameter, by the placing of a large central boulder and smaller rocks to mark the cardinal compass points and the direction of sunrise and sunset at the solstices. Many school parties and members of the public have already visited the site and heard a 'presentation'. We may hope that soon every school-child in eastern Massachusetts will have at least a smattering of astronomy.

An entertaining article called 'Digital Sundials—Time at your fingertips', by Fred Sawyer and Mario Arnaldi, recalls the time when the outstretched hands were used as an approximate measure of the time-of-day. A small stick, the length of the forefinger, was held between forefinger and thumb of the left hand, the outstretched hand was held flat and then turned at the wrist so that the palm faced to the right. Quoting from the translation of a Latin text of 1679, the authors write: 'If the shadow falls on the extremity of the forefinger, it is the hour of sunrise or sunset, the sixth hour......If on the extremity of the ring-finger, it is the 8th hour; if on the extremity of the little finger, it is the 9th, if on the third joint, it is the 12th or midday...' And so on. In a further development you could use a two-handed approach, with both thumbs and both forefingers. Better, (or more accurate) was a device for a nocturnal, using one of the stars (Phecda) of Ursa major and Polaris, and an imaginary line between them as the hour-hand of a clock, then comparing this with the fingers of your outstretched. hands. Nowadays, because of calendrical changes since this system was devised, the two pointer-stars of Ursa major lined with Polaris give a better hour-hand for this exercise. Here is something to try out in the garden on the next starry night.

Of particular interest to British readers is the last photograph in this issue, a full page photo of the trophy Dial for the Sawyer Dialing Prize, which was designed and made by Tony Moss of Lindisfarne Sundials in Northumberland. It is a magnificent equatorial in brass, adjustable for latitude and longitude, and carrying the logo of NASS, and also the motto ZHOI. These Greek letters, used on Greek sundials as the numerals for 12, 1, 2 and 3, form the Greek word meaning 'Live'. The Dialing Prize, inaugurated by the Sawyer Family and financed 50-50 by the Family and NASS, is to be presented annually to a person for 'contributions and dedication to dialing and the dialing community'. The award will include a cash prize to be donated to 'a dialing endeavor or non-profit organization of the recipient's choice'. The first such award, made in August 2000, was given to Fer J. deVries, of Eindhoven, The Netherlands.

COMPENDIUM, JOURNAL OF NASS: VOL.7. NO.4 (DECEMBER 2000)

In this issue we reach the final part of D.Collin's 4-instalment article on 'The Theory of a Vertical Bifilar Sundial'. We also have an article by F.J.deVries and others, on 'Multiple Analemmatic Sundials' : how to lay out analemmatic dials on the 5 exposed faces of a cube or of any 3-dimensional object. Next comes an article by Ferrari on 'A curious property of bifilar sundials', which goes on to describe the layout of a double-horizontal bifilar dial, and then a tri-filar vertical. So we reach Page 16 of this 32-page issue before we find a photograph of a real sundial; so far we have just had pages of diagrams and line-drawings. That is not quite fair: there is a photograph of bifilar on the front cover. But am I the only reader to wonder whether anyone actually carries out, and realises in practical terms, all the ingenious mathematical designs of which *Compendium* is full? And whether, if these designs are created, they are of any aesthetic value or interest? Or do NASS diallists carry on designing dials just for the fun of the mathematical challenge?

A regular feature of the *Compendium* is the Dialing Quiz. These problems are usually set by Fred Sawyer, and readers are becoming familiar with Nicole, the young woman heroine of the story-lines accompanying the Quiz. But a recent Quiz was set by Gianni Ferrari of Modena, who asked readers to tell him the declination of the wall which he must build for a vertical dial, a birthday gift for his wife whose birthday is 28th July, the latitude and the sun's declination for that day being given: and maximum number of hours of the dial's surface to be illuminated.

Another regular feature is the sections called 'Sightings' and in this issue there was a photograph of a gravestone ornamented with a rather inaccurately carved vertical dial, a symbol of mortality replacing the more usual willow-trees, urns and skulls. This was found in a collection of gravestones removed from a cemetery and put into the crypt of a church, when this church was newly built in 1814 on the site of part of the cemetery. The gravestone, commemorating one John Trowbridge who died in 1749, has been stored in the crypt for 180 years away from acid rain and the weather, and looks much as it must have done 1814

Two or three pages of each issue are devoted to short paragraphs under the general heading 'Letters, Notes, e-mail, Internet', and there are often some jewels in this random collection. One item in the December issue made a claim that the letter-writer had visited the world's most northerly sundial, a handsome dial of wooden-carved hour-markers, in the north of Iceland, Lat. 65° 40'; a couple of photos were included. Forthcoming or recently published sundial books get a mention in this section. So too is the good news that the 'Sunwheel' at Amherst, MA, whose start was recorded in the September issue, has benefited by a week of fine weather during the placing of 112,000 pounds of granite and 102,000 pounds of crushed stone, and is now complete.

M.S.

READERS' LETTERS

THE SUN-MOON PARADOX

This title refers to the fact that the line connecting the two horns of the moon's sickle is not perpendicular to the line between the sun and the moon as we see them in the evening sky near sunset. This is very evident when the moon is "young".

This paradox has a very simple explanation. The sun as we see it seems to be situated on the same celestial hemisphere as the moon. We therefore conclude that the distance earth-sun is equal to the distance earth-moon, and we are surprised that the moon's sickle does not "1ook" towards the sun.

But in reality the distance earth-sun is ca. 400 times the distance earth-moon. Therefore the sunrays sun-moon run almost parallel with the sunrays sun-earth, which explains the position of the moon sickle we see.

This is very simple, though some people think differently. In 1937 the Dutch professor of astronomy Minnaert wrote a book which was translated into English under the title: "Light & Colour in the open air." On page 152 he explains the above phenomenon in these words:

> "Immediately related to this is the observation that the line connecting the horns of the moon, between its first quarter and full moon, for instance, does not appear to be at all perpendicular to the direction from sun to moon; we apparently think of this direction as being a curved line. Fix the direction by stretching a piece. of string taut in front of your eye; however unlikely it may have seemed at first, you will now see that the condition of perpendicularity is satisfied."

If this were true, the moon's sickle turns over towards the sun by the simple application of this string, but it regains its normal position when the string is removed. At the same time it changes from a small sickle into a full first quarter and back.

Strange as this may seem I find it even stranger that in June of 1999 Jean Meeus wrote an article in the Belgian "HEELAL", using the same method. The only difference is that he replaced the taut string by a broomstick. This smells of witchcraft and in my opinion it is much better suited to the occasion.

But the reason I wrote this article is that the same "explanation" of the paradox crops up after 62 years, and I am anxious to know if anyone can give me some information on an earlier publication or on the birth of this string-cum-broomstick conjurer's trick.

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'ELEVEN DAYS'

In her article "A Brief History of the Measurement of Time (March 2001) Harriet Wynter writes that at the time of the changeover in this country from the Julian to the Gregorian calendar "there were riots in the streets, 'Give us back our eleven days' cried the mob."

I think that to say there were riots is more folk-myth than history. Dr. E.G. Richards in his book 'Mapping Time' (Oxford University Press paperback, 1999) states: "There were said to have been riots in Bristol but recent attempts to find reports of these in contemporary newspapers have drawn a blank " (p.255)

I also take another point from Dr Ricbards' book that is overlooked in this context. The Act of Parliament (24 Geo II, ch. 23) ordained that apart from dropping the famous eleven days, the year 1751 which began on 25th March (Old Style) should end on the last day of December and the next day, 1st January, should be the first day of 1752. No one seems to have objected to this.

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LATITUDE PROBLEM

Can anybody help?

John Davis and I have been in correspondence about latitudes engraved on dials and the fact that they very rarely agree with their location latitudes. A quick survey from the Register revealed several engraved with a latitude given to an arcminute, but the dials are not there even if the provenance is quite good, like being a vertical dial firmly attached to a church.

John Speed's map of Scotland (*1605*) has latitudes differing by over a minute from today's O.S. values.

Is there anyone who knows about old maps and whether latitude delineation has altered greatly over the last two or three centuries?

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WITHDRAWAL OF ARTICLE

I regret that, due to unforeseen circumstances, Part 2 of my '*Brief History of the Measurement of Time*' will not be printed in the Bulletin. For those who were interested in Part 1, I append a list of my references and further reading for the whole article. This was to have appeared following Part 2.

References and Further Reading

Having embarked on a brief history I have endeavoured to keep it short. However, it has not been a simple task, for in pursuit of brevity I have left a great deal out. Therefore, for those who wish to know more, here is a minimal bibliography.

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'WHAT'S THE ANGLE ?' (1)

I was delighted by Mr Capon's letter to the March Bulletin, about the best distance from which to view a vertical dial. His mathematical and historical exploration of the problem is both interesting and satisfying.

In practice, a couple of things are worth noting. Firstly, pleasing though it is to have the precise distance, the answer in every-day terms is of course to stand away a distance of H + D/2, so that the centre of the dial is at an elevation of 45°, as Mr Capon clearly shows in his Fig 2.

Secondly, when taking a photograph, the solution depends on one's equipment. Take the fairly common case of a two foot dial on a church wall at a height of about twelve feet. My eye level is at six feet (and I use a 70mm - 200mm zoom lens on a 35mm camera). For viewing, therefore, I should stand about seven feet away ((12 - 6) + 2/2), or 6' 11" by Mr Capon's formula. The situation is different, however, for photography. From seven feet away, I could fill my frame with the dial on an intermediate zoom, though at 45° it would be pretty distorted by perspective. Far better to move back to fifteen feet with maximum zoom, and get a less distorted picture, still filling the frame. The rule in practice is to use your longest lens, and get as far away as you can, while still filling the frame. Yes, I know this ignores problems of the camera shake and shallow depth of field with long lenses, but at some point instinct and common sense will take over to give a good result.

Incidentally provided the elevation is not too great, the photographing distance (in feet) is approximately (dial vertical dimension in feet) multiplied by (zoom length in inches). This is really the 'through the air' distance, and should be reduced accordingly for the 'on the ground' distance, for steep elevations. The rule depends on the fact that a 35mm negative is about one inch high (actually 24mm). It does not work therefore for larger or smaller format cameras.

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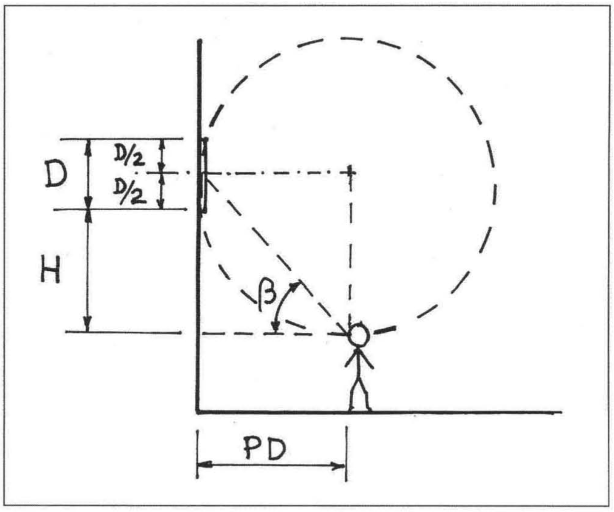
'WHAT'S THE ANGLE ?' (2)

In Bulletin 13(i) March 2001, Mr. A. Capon gives a mathematically correct equation., for calculating the optimum distance (PD) at which to photograph a dial mounted high on a wall. I would like to propose a simpler equation.

Mr. Capon's equation is $PD = \sqrt{DH + H^2}$ which can be re-written as $PD = \sqrt{H.D + H}$. This PD is the geometric mean of two measurements--the heights above eye level of the bottom and top edges of the dial plate. In many situations the geometric mean will not be significantly different from the arithmetical mean, which is $\frac{1}{2}[H + (D + H)]$ i.e. $(H + D/2)$, which is the height above eye-level of the centre of the dial. It can be seen from the enclosed figure (based on Mr. Capon's Fig.4) that if PD is equal to (H + D/2) then the line from the observer's eye to the centre of the dial rises at 45° to the horizontal. So all the photographer needs to do is to locate the position at which the camera is tilted at 45° when the dial centre is in the centre of the viewfinder. A piece of card cut to a right-angled triangle with 45° angles might be a useful guide.

Optimum distances calculated from the two equations, for dials 0.2, 1, and 2 metres in size, and for two extreme values of height H, are summarised in the table. It can be seen that the differences are insignificant, especially if the distance is marked out by pacing. When the height H is very small, both equations give absurdly short distances for PD. But in this situation, when the bottom edge of the dial is near eye-level, you don't need an equation: you just use the viewfinder



The alternative good position: when PD = (H + D/2), angle is 45°

Table: Comparison of calculated values of PD

D	H	(D + H)	H(D + H) (Capon)	H + D/2 (Head)
0.2	2	2.2	2.098	2.100
0.2	10	10.2	10.099	10.100
1	2	3	2.449	2.500
1	10	11	10.488	10.500
2	2	4	2.828	3.000
2	10	12	10.954	1.000

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A STARDIAL

SVEN OLOF LARSSON

INTRODUCTION

It is fascinating that a clock is an analogue to the sun's position on the sky. If we ignore the fact that the hour hand rotates twice a day, clocks can be seen as miniature models of the sun-earth relation. People sometimes ask what is the point of using a sundial when you have a wristwatch, and I often find it difficult to find a cogent answer to this question. It could partly depend on people's different views. Many probably do not recognise that the sun is the fundamental basis for our timekeeping. The sun made life possible on earth and the atoms we are made of have been created in supernovae. Exploded stars have given birth to our existence. We often take the regularity of the heavens for granted and we naturally feel tired in the evening and sleep in the night. Imagine that the orbit and rotation of the earth had been different: life, if it existed, would probably also be very different.

Considering the fact that solar time and local standard time (clock time) differ, it is after all easy to use the Equation of Time (EoT) and longitude correction to cope with this. Having some kind of device, which preferably focuses the sunbeams, the exact time of day (within fractions of a second) can be told. I guess that many of you do not adjust the wristwatch to make it accurate to the second. The hobby of sundial construction, as I see it, should have simplicity as a guiding-star together with accuracy, design, and originality. It has been a dream of mine for about 15 years to build a sundial that is simple, yet accurate to at least a minute. By using a movable arm to enable exact reading, automatic conversion from local solar time to the time of the clock has been made possible. The arm can also be used to aim at a bright star, consequently telling the time at night as well. This is the reason why I do not call the construction a sundial because any of the brightest stars can be used.

THE DIFFERENT PARTS

In what follows, numbers refers to the positions as indicated in Fig. 1. The stardial is basically an equatorial sundial. Instead of an ordinary style made of a thin rod (possibly with a small sphere on it) a projection plate (1) is used. An analemma[1] is drawn on this (see Fig. 2). When reading the time, the projection plate has to be rotated correctly. When the sun shines a shadow of the curve of the analemma is projected on a cross (2) at the end of the shadow arm (3). A prop (4) is used to fix the arm's position. Attached on the axis (5) (which is parallel to the polar axis) is a time hand (6) with minutes engraved on it. The hour marks below the time hand forms the dial face (7). With the stardial one has the possibility of telling the time in the night with help of a simplified astrolabe (8) that is put inside the hour marks. The whole furniture and the astrolabe are shown in Fig. 3. The dimension is approximately 8x8x12 inches (20x20x30 cm). It is made out of aluminium, plexiglass, brass, wood, and transparencies. To read the time one rotates the projection plate together with the shadow arm so that the shadow line from the analemma falls right on the cross. The time is read from the dial face.

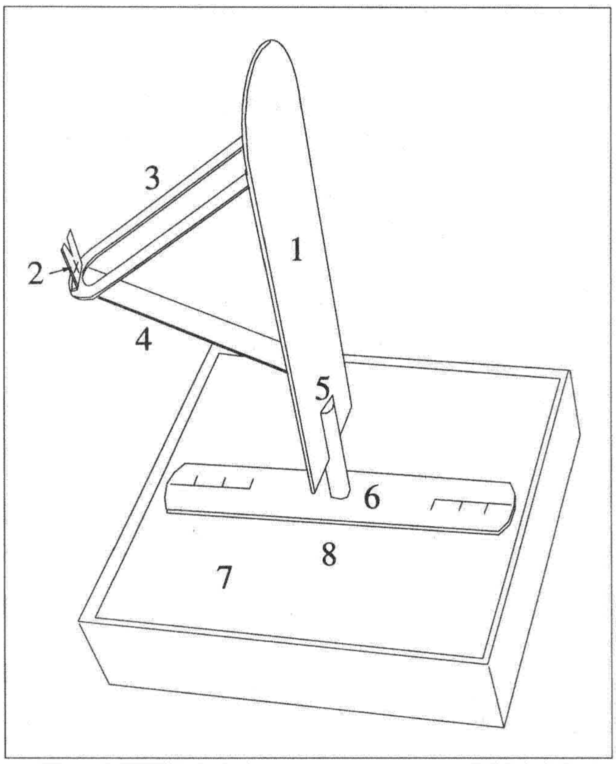

Fig. 1. Different parts of the stardial

The time hand is a kind of rule that has 30 minutes engraved on it along a line. This line will cross one of the slightly curved half-hour lines on the dial face. The crossing point indicates the time. One of the two lines of the shadow (seen horizontally in Fig. 2) can be made to fall on the cross by rotating the shadow arm. The cross is positioned normal to the centre of the projection plate. The time hand will tell the direction of a line from the middle of the projection plate to the cross on the shadow arm. However, correction for the difference to the local time zone meridian can preferably be made here. Since the shadow lines from the analemma deviate from a vertical line corresponding to where the traditional style should be placed (dividing the analemma into two halves), the time hand will be automatically moved to show the time of the clock when the shadow line falls on the cross.

Unfortunately, one has to select which shadow line to use (right or left). The lines have been made so that one is black (opaque) and the other is transparent making a bright line when the sun shines through it. The dark shadow line is to be used for the first half-year, and the bright line for the rest of the year. To be able to make a bright line the surrounding area on the plate is made black, resulting in a division of the plate into four quadrants. Two of them, let us say the lower left and upper right are black. There is also a dotted line marking the vertical centre where the usual style is placed.

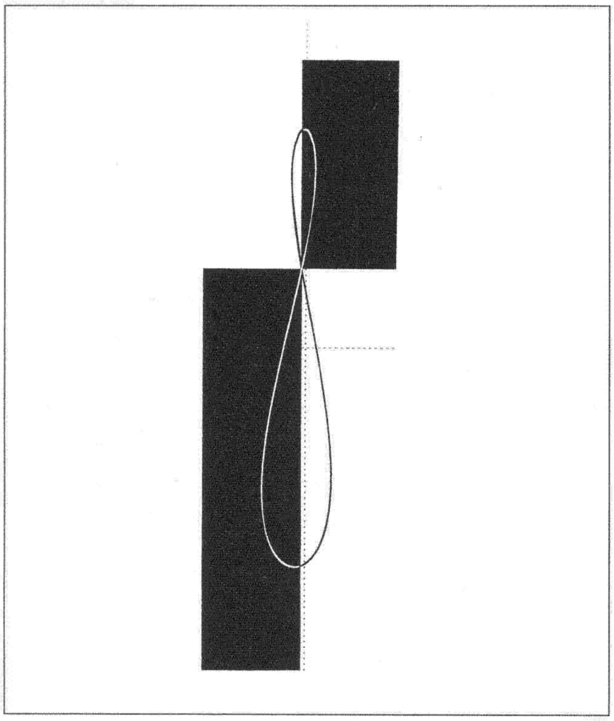

Fig. 2. The analemma on the projection plate

THE DIAL FACE

Instead of a ring with time marks (as used on a traditional equatorial sundial) the time scale has been moved to the bottom of the dial. Here, on a plate perpendicular to the axis we have the dial face, and just above it the time hand (attached to the axis). The axis, which penetrates this plate, is fixed to the hole of a ball bearing attached right under the plate. The construction of the dial face is as follows (see Fig. 3). For each half-hour a slightly curved line is plotted from an inner circle to an outer circle on the face. Looking on the time hand when it is rotated around the axis, only one line at a time will cross the time hand somewhere along the engravings of the minutes. At the point where a line finishes at the outer circle a new line starts from the inner circle where the scale on the time hand is marked with zero/thirty minutes. The time is found where the time hand crosses one of these lines. As far as I know, this method was invented by the Swedish-Danish astronomer Tycho Brahe (1546-1601). The hours are marked besides each alternate line.

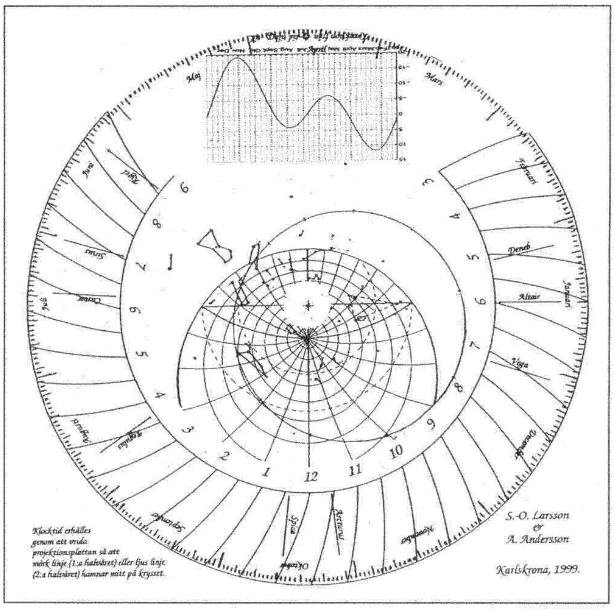

Fig. 3. Clock-face and astrolabe

THE ASTROLABE

The astrolabe is used to read the time at night. It is transparent and placed inside (and above) the dial face. The astrolabe has in this case the same function as a nocturnal. One rotates it so that the positions of the stars on it show the directions to these in the sky. The use of an astrolabe makes it possible to see the altitude and azimuth of the sun and the stars, and the length of dawn and dusk can be found for any date of the year. The time for a particular star to rise or the time for it to be in due south can also be told just as with an ordinary astrolabe. It is made up of a so-called tympanum, which is a couple of circles for the azimuths that show the bearings and almucantars that show the altitudes. It is made for a specific latitude and it shows the heaven from the

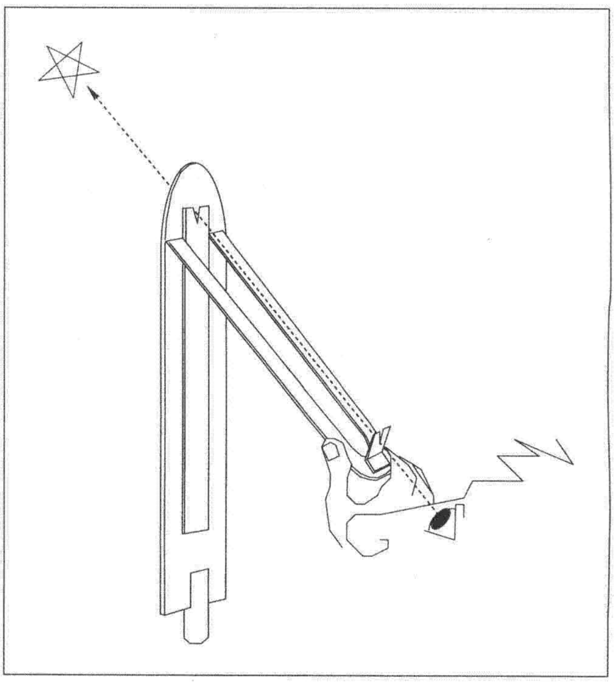

Fig. 4. Aiming at a star

north celestial pole down to the tropic of Capricorn. On top of it and on top of the time hand the so-called rete (Latin for web) is placed. The rete shows the brightest stars together with the ecliptic. At its periphery the days of the year are marked. Placing the time hand at a date, the position of the sun on the ecliptic is easily found with the help of the time hand. The rete can be rotated freely around the axis. The middle point, where the axis is, corresponds to Polaris. It should be noted that the time hand corresponds to the rule on an ordinary astrolabe. On the tympanum there is also a circle (called twilight line) marking an altitude of 6 degrees below the horizon. This is used to see the duration of the dusk and dawn. There is also the possibility of getting a rough estimate of the time after sunset and until the stars becomes visible by placing the point on the ecliptic ("where the sun is") on an estimated altitude below the horizon.

The shadow arm is movable up- and down-wards. When used in the day the prop is used to fix the position. To tell the time in the night, the prop is detached from the arm and raised to an upright position and fixed just over the projection plate. A slit is found above the cross on the shadow arm and a line is drawn on the top of the dial. These two sights are used to aim at a bright star as sketched in Fig. 4. The rete is rotated so that the actual star is placed on the middle line of the time hand. Fixing the rete with one hand, the time hand is then rotated to the date in question (seen at the border of the rete). The time is read as usual.

It is easy to check the accuracy of the dial by reference to the time of sunrise or sunset since these times usually are easy to find tabulated. The time hand is placed on the actual date to find the position of the sun on the ecliptic. The rete is rotated together with the time hand so that the sun gets positioned on top of the horizon line. (The line of the horizon is actually 0.83 degrees below the horizon to compensate for the bending of light through the atmosphere.) Placing the time hand over this point on the horizon gives the time. A correction table placed inside the hour marks and beside the tympanum, compensates for the EoT.

REFRACTION

Refraction is the phenomenon of bending of light, in this case, through the atmosphere. Ordinarily, this will not cause much error for a sundial. When trying to get an accurate reading, one has to take this into account. The "lifting" of the sun at sunset is about 0.57 degrees. The resulting error in the time reading is the component of the lift along the equatorial plane. The error can be plotted as lines on the astrolabe, e.g. for 5, 10, 15 seconds and so on. The error is then corrected by adding the error in the afternoon or subtracting it in the morning. However, to get an accurate measurement, the sun should be at least 6 degrees over the horizon. The greatest error on latitude 56° is at sunset/sunrise when the sun is due west/east, and is about 1 minute and 16 seconds (sin(90-56)*0.57*4 minutes). The refraction error is zero for measurements in due south/north, i.e. a very low altitude star in the south has an error only in the altitude, not in the horizontal direction. For a latitude of about 45-55 degrees one can say, as a rule of thumb, that the error is about 10 seconds when the sun's altitude is 10 degrees.

USING THE MOON

It is tempting to use the moon for measuring the time. However, since the orbit of the moon is so elliptic, it makes this very difficult. Even when given a table beside the sundial with the time-angle for the sun, one has to know the number of days since the last new moon. If you can see a shadow cast by the moon and thereby also are able to see a time indication on the dial, the hours and minutes from the table should be added to get the time. Even if the table is adjusted to a month of 29.53 days corresponding to the synodical month (phase-to-phase), it is not useful. Besides the variation in the synodical month, the time from half moon to new moon will vary much more. This will give an error of about one hour even if additional tables are given for the exact time since new moon and corrections for the always-changing time angle.

Another way of using the moon has been adopted for the stardial. The moon can be used for estimating the direction to the sun. This method can be used to rotate the rete quickly into place when perhaps stars are hard to see in the moonlight or in the presence of disturbing lights. The following description refers to Fig. 5. One looks at the moon so that it can be seen in the extension of the axis of the stardial, which is parallel to the polar axis (A). The phase of the moon is observed along a line that is perpendicular to the axis (B) and the proportion of the light and darkness is noticed. Half of the axis on the stardial is painted white on the front side. After rotating the stardial so that the painted fields have the same proportion of light and darkness (C), the side of the projection plate that is to be faced to the sun in the day will now face the sun even if it is under the horizon. The time hand will show the direction to the sun and the time can be found. The clock time is obtained by adding the minutes from the correction table. Furthermore, if the rete is rotated so that the actual date gets positioned at the time hand, the orientation of the rete corresponds to the heaven, and a convenient star for a more accurate time measurement can be found more easily.

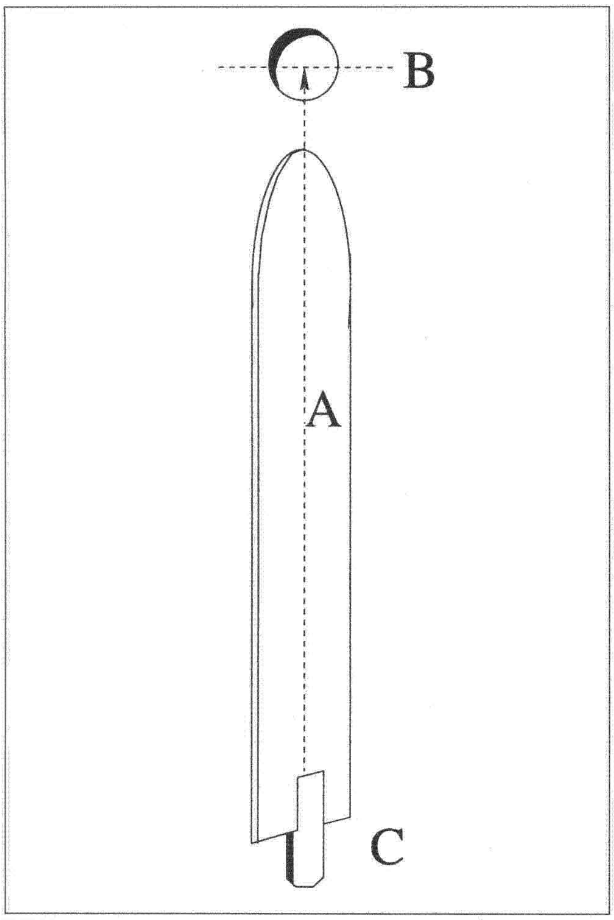

Fig. 5. Using the phase of the moon

CONCLUSIONS

A prototype of the stardial was made in 1998-99 to evaluate the precision (see Fig. 6). An error less than 15 seconds is normal for this device. We have not evaluated the error at night, but at a first test, it seemed to be less than a minute. At the days around the winter- and summer-solstices, a special way of obtaining the clock time should be used because the shadow of the analemma is practically horizontal during these days. The prop is detached from the arm and raised to an upright position. A hole in the prop will give a sun spot over the cross and subsequently the solar time is given. The clock time is found by using the correction table.

On a cold morning it was found that small water drops had condensed below the protective plexiglass over the furniture. Ventilation holes in the corners were made to decrease this effect. We found that the projection plate also suffered from condensed water. A better design would be to turn the stardial 180 degrees and engrave the analemma on a solid plate (e.g. aluminium). The cross at the end of the shadow arm would instead be a hole (aperture), for the sun beams to path through. One should also note that the rete had frozen to the plexiglass in the winter but was easily detached. It is also important to select stainless materials, which we failed to do for some of the screws and nuts. All wood should be avoided, not only because it has a tendency to rot after some time but also because of the contraction or expansion caused by changes in the temperature.

Fig. 6. Photo of the Stardial

Finally, I have really enjoyed the work and I am more than satisfied with the result. However, the time estimation obtained when using the moon has not yet been evaluated.

ACKNOWLEDGMENT

The author wishes to thank Mr. A. Andersson for the fruitful discussions and cooperation when designing and building the stardial. The stardial is placed in the middle of his herb garden.

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FROM WAUGH TO WEIR: EASY ANALEMMATICS

MICHAEL HICKMAN

INTRODUCTION

Waugh[1] Mayall and Mayall[2] and Cousins[3] lead us to believe that the design of analemmatic dials is difficult and that it involves tedious calculations. However there is a method of designing such dials involving no maths at all except possibly for enlarging or reducing the design to the required size. Even that may not necessitate calculations as many modern photocopiers are capable of enlarging or reducing image size.[4]

In a feature on sun compasses that I wrote for the Bulletin some years ago[5] I mentioned the Cole sun compass and commented on its similarity to a navigational publication known as Weir's Azimuth Diagram[6]

At this point I should fall upon my sword, as there, staring me in the face, was a universal design for analemmatic dials and I didn't recognise it. Also in my feature I outlined the maths of the Cole sun compass and I have only six years later realised their identity with those of Weir's diagram

WEIR AND SUNDIALS

I suddenly saw that the diagram, with a suitable gnomon, could be used without any modification as an analemmatic dial in any given latitude from the equator to 65° north or south; and so I spread my copy out on a table indoors in the sun and aligned it to the north—south line

With a cocktail stick held vertically in Plasticine at the appropriate declination point there was my analemmatic dial. With it I could read local apparent time to within a minute and hence determine GMT or BST.

If I didn't want to use the diagram itself as a dial I could, knowing my latitude, trace or measure the required points from the diagram and use those as the design for a dial and by suitable scaling that dial could be of any size. The diagram also included the scale in its centre for declination.

Let us look at Weir's diagram in more detail. This will be virtually non-mathematical; those who are interested in the maths are invited to read the appendix to this article. They will also find mathematical treatments in early editions of navigation textbooks[7] though modern editions do not appear to handle this topic.

Navigational users of the diagram know their latitude, the declination of the sun and the local hour angle. Armed with these they can easily determine the sun's azimuth.

Conversely if we know our latitude, the sun's declination and its azimuth, we can and do determine the local hour angle and hence the local apparent time. From that we can determine GMT or BST as we wish.

For copyright reasons (and because there is much detail that would be lost) it is not possible to show a reduced picture of Weir's diagram in the Bulletin. However I and my trusty spreadsheet have calculated the curves for latitudes 40° 50°and 60° and for local apparent times of 9, 10, 11, 12, 1, 2 and 3 o'clock and I have included part of the declination scale.

These curves are shown in Figure 1. For ease of setting up my spreadsheet and producing the Figure I have shown curves only for hour angles of 270° through 00° to 90° and I have shown only northern declination. Note that Weir's diagram gives the direction of the sun whereas we are interested in its reciprocal, the direction of the gnomon's shadow. Thus the times in Weir need to be altered by twelve hours for dial design. (Weir's diagram consists essentially of Figure 1 together with a mirror image of that figure about the x axis. Thus in Weir what I have shown as semi-ellipses become full ellipses, the hyperbolae are reflected about the x-axis and the declination scale caters for southern declinations as well.)

It should be noted that the declination scale is common to all latitudes, unlike those resulting from Waugh's design. This is done in azimuth diagrams by dividing the three parameters of the dial design (horizontal distance H in Waugh, vertical distance V in Waugh, and declination, Z in Waugh) by cos ϕ where ϕ is the latitude. This division alters only the scaling of the diagram but has no effect on the angles therein for any particular latitude.

The quotients are then plotted as two sets of curves, one keeping latitude constant and the other keeping the local apparent time constant The first set will comprise ellipses and the second set will comprise hyperbolae as in the foregoing diagram. The intersections of the hyperbolae with the ellipse appropriate for the latitude will give the locations of the points on our analemmatic dial and the centre scale will give the position of our vertical gnomon according to the declination.

This scale can be marked to show either the actual declinations or the relevant dates as in Waugh's design. That is perhaps preferable as it saves users having to look up the declination for the day when they wish to use the dial.

DIAL DESIGN

How then to use Weir's diagram for dial design?

First get the diagram. This is available as hydrographic chart 5000 available for about £8 from yacht chandlers and other nautical dealers.

Next get some tracing paper or a ruler and protractor.

Each hour point's location will be at the intersection of the appropriate latitude ellipse and hour angle hyperbola. Trace these and the declination scale and there is your analemmatic dial design.

If you are not going to take account of your longitude in your design then it will be symmetrical about its vertical axis and so A4 paper will do. You can trace the hour points for the left hand half of the dial and then reverse the paper to give you the design for the right hand side. However if you wish to take account of longitude then, unless the dial is on the Greenwich meridian it will not be symmetrical and so A3 paper will be required (or two A4 sheets joined).

As an alternative to tracing the design you could measure the distance of the point from the origin and the angle from north-south of the line between it and the origin. So long as the angles remain constant and distances are multiplied by a common number it would then be easy to design a dial of any required size to suit any site from your pocket to your garden.

Unless you take account of longitude, using Weir will give you a dial that shows local apparent time. If for example you are 2° west of Greenwich then local apparent time will be 8 minutes behind that at Greenwich. This is easily compensated for using Weir's diagram; your marker for say 11 a.m. will actually be on the hyperbola for 10.52 a.m. and similarly for other times. Also the hyperbolae are helpfully marked not only by degrees but also by 4 minute intervals and interpolation between them is not difficult.

If your latitude is not an exact number of degrees then again interpolation may be used, this time between the latitude ellipses which are at 1° intervals.

You can design your dial to show GMT or BST as you please but I can't think of any way in which to allow for the equation of time. Any suggestions on this aspect will be welcome.

CONCLUSION

I have not described the actual use of analemmatic dials as that is straightforward and already well covered in the existing literature. However I believe that this method of designing such dials has not been suggested before. I hope that it will appeal to you and that I have shown you that analemmatic dials can be designed using little or no maths and certainly no trigonometry.

Have fun and do let me know how you get on.

APPENDIX

Waugh gives the following:

Horizontal distance H of the hour point from the origin O $= \sin t$

where t is the appropriate hour angle

Vertical distance V of the hour point from the origin O $= \sin \phi \times \cos t$

where ϕ is the latitude

Distance Z of the gnomon from the origin O on the declination scale $= \tan dec \times \cos \phi$

where dec is the declination.

Divide Waugh's expressions for H,V and Z by $\cos \phi$. This will not affect the resulting azimuth of the sun but only the size of the diagram.

Calling the resulting functions H', V' and Z' we get

$$H' = \sin t / \cos \phi$$
$$V' = \cos t \times \sin \phi / \cos \phi$$
$$= \cos t \times \tan \phi$$
$$Z' = \tan dec$$

Thus the declination scale is no longer a function of latitude but only of declination, latitude being taken into account in deriving H' and V'.

If we eliminate t between H' and V' we get curves dependant on only latitude ϕ and these will be ellipses. The equations of these curves may be shown to be

$$H'^2/\sec^2 \phi + V'^2/\tan^2 \phi = 1$$

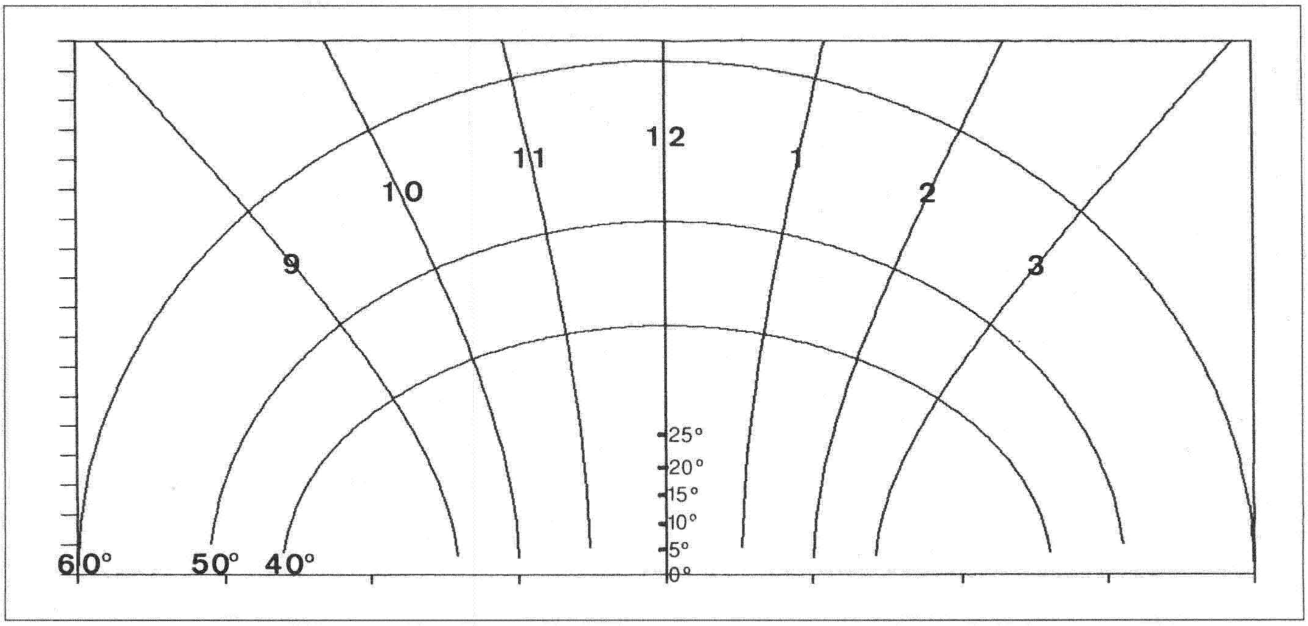

Fig. 1. Latitude ellipses, hour angle hyperbolae and declination scale (upper halves of each only)

If we eliminate ϕ between H' and V' we get curves dependant on only t and these will be hyperbolae. Their equations may be shown to be

$$H'^2/\sin^2 t - V'^2/\cos^2 t = 1$$

These curves intersect at what Waugh calls the hour points and their combination, with the inclusion of the declination scale, is Weir's azimuth diagram.

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4 Informal discussion Hickman/County Office Services Dorchester

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6 *Modified Weir azimuth diagram Hydrographic chart 5000* Admiralty 1940 and later

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DATING A SUNDIAL BY CALENDAR CHANGE

MIKE COWHAM

The change from Julian to Gregorian calendars can often be used as an aid in the dating of sundials and related instruments.

The calendar change was made at different dates, depending on the country involved. The Catholic countries changed almost immediately in 1583, following the edict of Pope Gregory XIII. The Protestant countries delayed implementation of this new 'Papal Calendar' as long as possible, not wanting to agree with anything remotely Catholic. In the case of Great Britain and its colonies, including the Americas, the change was finally made in 1752.

HOW DOES THIS HELP TO DATE A SUNDIAL?

The peak period in our history for the making of fine sundials stretched from around 1680 through to 1850, by which time communications had improved and the sundial was rapidly becoming superseded. There was the electric telegraph, and later radio, so that every town, village or stately home no longer needed its own time reference.

The method described below will work only where calendar or equation of time information is provided for the dial.

Before the calendar change, the vernal equinox fell on (or around) 10 March and the corresponding autumnal equinox around 10 September.

The correction removed 11 days from the British calendar, so that 2 September 1752 was followed by 13 September. By this means, the Vernal Equinox was reset to its traditional point of 21 March, the 'First Point of Aries'.

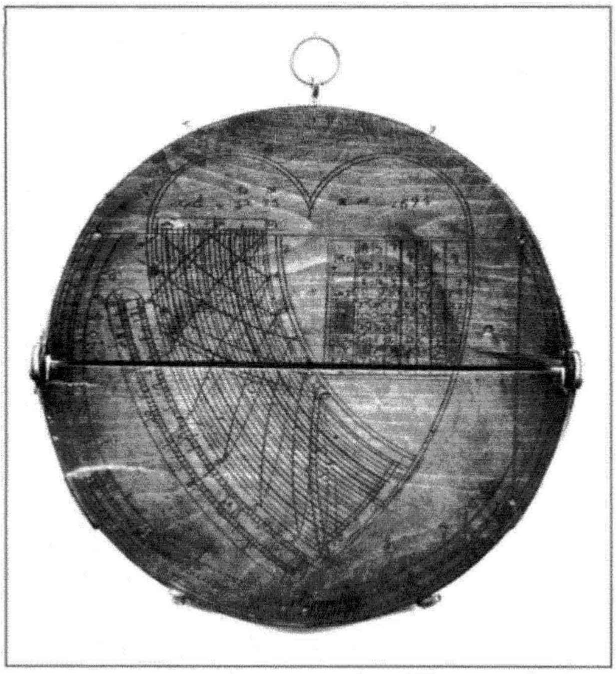

Fig. 1a. English Quadrant in the style of John Browne, dated 1694

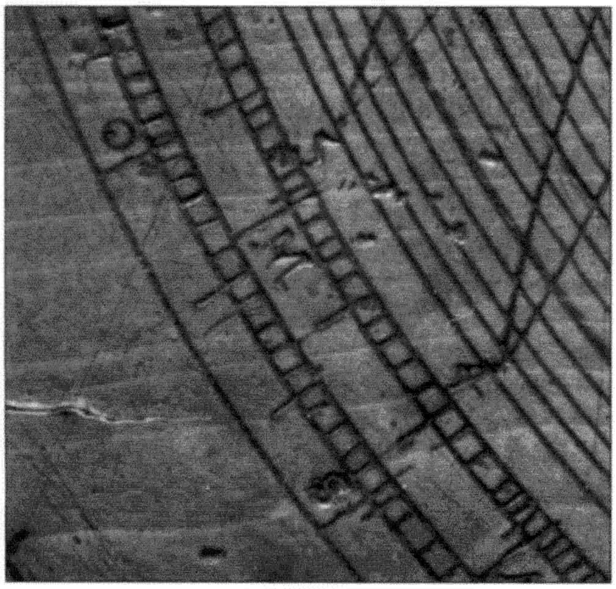

Fig. 1b. Quadrant scale enlarged

In the case of an altitude dial, a calendar scale is essential for setting it. Close examination of the point where the two scales, vernal and autumnal cross will show if the dial was made before or after the calendar change. Naturally, armed with a list of dates where this important change was made, it is possible to use this method for the dials of several countries.

Fig. 1. and Fig. 2. show the calendar scales on two English wooden quadrants. Fig. 1. shows that March 12 is opposite September 12. This is clearly a pre-1752 quadrant. Actually it has been conveniently dated 1694 by its maker. Fig. 2. shows a similar quadrant where March 22 is opposite September 22. This was made after the calendar reform, being later than 1752, but not by many years, as this type of quadrant was already becoming obsolete.

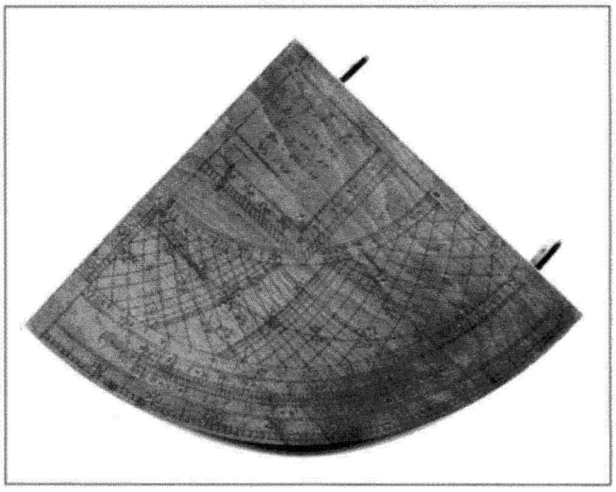

Fig. 2a. English Quadrant, second half of 18 Century

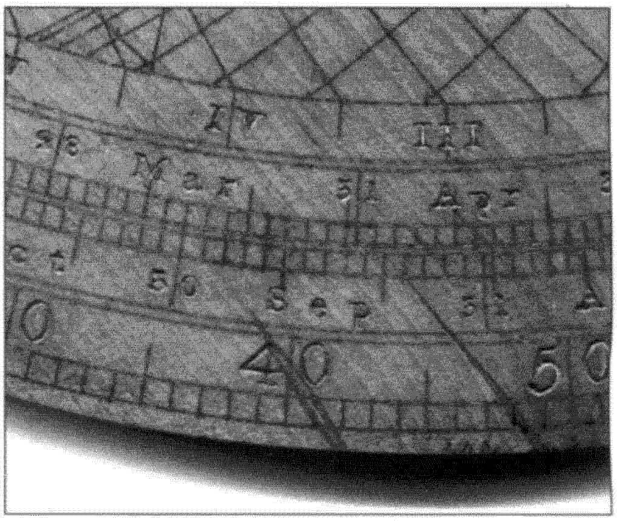

Fig. 2b. Quadrant scale enlarged

Using equation of time tables, similar checks can be made. A good reference point for EOT is when the correction changes polarity. i.e., when the EOT is zero. This happens four times each year, and these are times of maximum change - hence maximum sensitivity to calendar change. Avoid using the peak corrections, as the peaks stretch over a week or more, and we are looking for differences of only 11 days. Also check that the tables do not include a correction for the longitude, but this was not commonplace until universal time was adopted in the late 1800s. Therefore, it is likely to apply only to dials from Russia, China and Japan plus a few of the smaller European countries that had still to correct their calendars.

Fig. 3. is a portable dial by Thomas Wright. The EOT scale here shows one of the transitions on 20 August. Waugh[1] shows that this transition is now 2 September, which is actually 12 days later. Ignore the odd day or so of error, as the EOT can vary by this amount, depending on the part of the Leap Year cycle from which it is taken. (Strictly speaking, the Leap Year cycle is not only governed by the common 4 year period, but also the lack of a Leap Year on the Century date, except in every 400th year. Therefore the true cycle is 400 years.) This dial can be confidently dated before 1752.

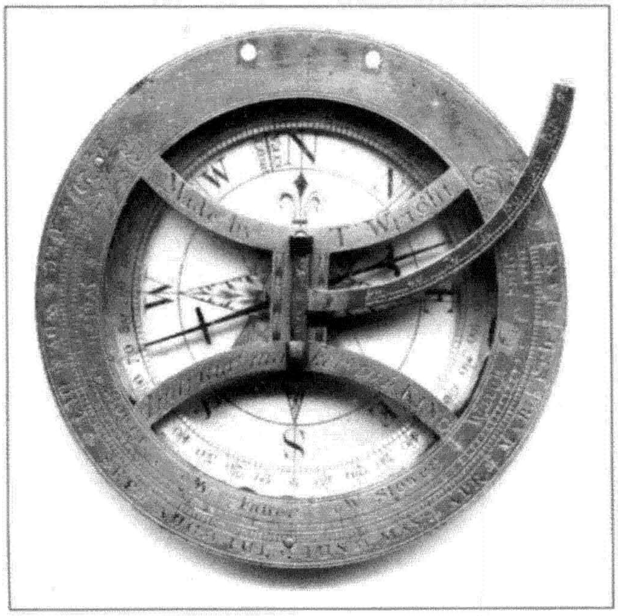

Fig. 3a. Dial Plate of Mechanical Equinoctial Dial by Thomas Wright, c1720

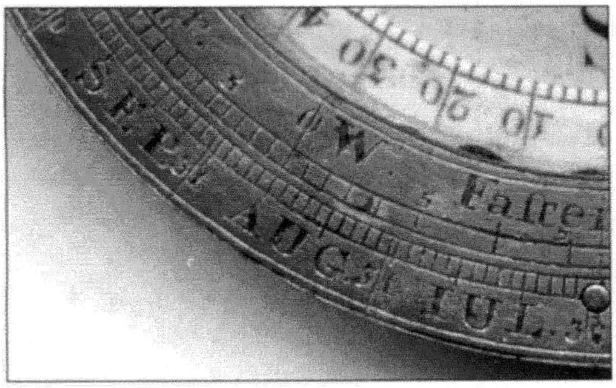

Fig. 3b. Calendar scale enlarged

Another dial by W & S Jones, Fig. 4., has a table of corrections engraved upon it. For convenience, taking the same transition point, we find that it gives a date between 31 August and 3 September. This dial is therefore later than 1752.

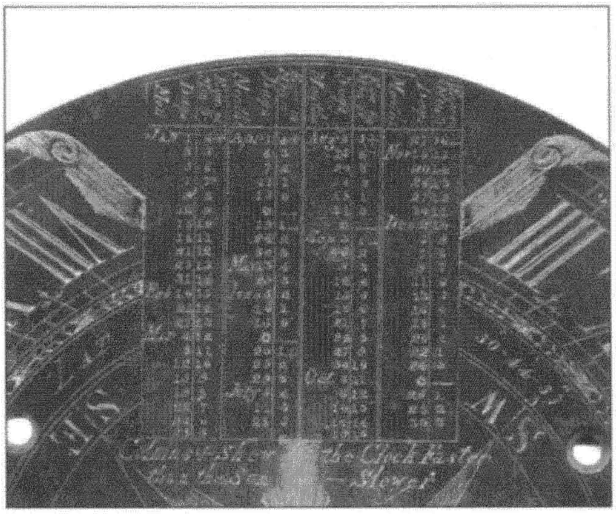

Fig. 4a. Garden Dial by W & S Jones, c1820

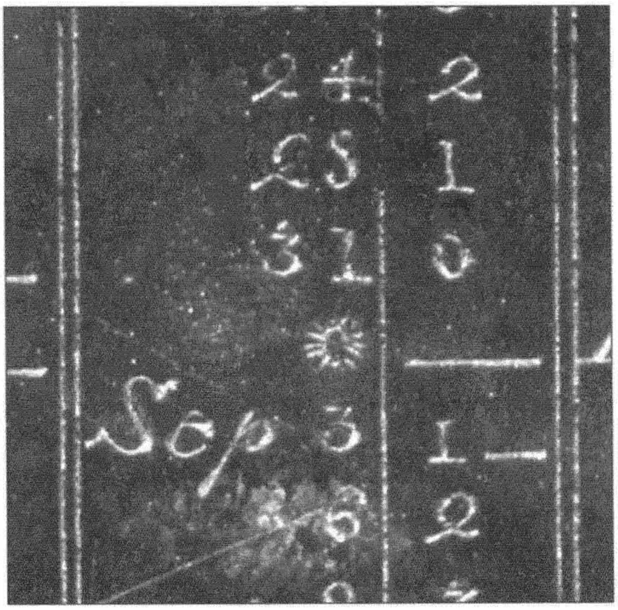

Fig. 4b. Enlarged EOT table

In Britain, the calendar change has been shown as a useful aid to dating old dials. The calendar change at different times led to much confusion across Europe. Spare a sympathetic thought for Germany, where the Catholic states changed to the new calendar in 1573 but the Protestant states retained the old one right up to 1700! Sundials and Perpetual Calendar devices made in Germany usually had both calendars marked on them. With the two calendars running simultaneously, the different churches could celebrate Easter up to two weeks apart.

For more information about the calendar, the method of finding Easter and the problems involved with the change, refer to the following works: -

D. E. Duncan: *The Calendar.* Fourth Estate, London, 1998.

M. J. Cowham: 'Calendar Systems and Perpetual Calendars' *Bull. Scientific Instrument Soc.* 62, 20-23 (1999), 63, 11-16 (1999), 64. 7-12 (2000).

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1. A. E. Waugh: Sundials, Their Theory and Construction. Dover, New York, 1973.

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THE LAID DIAL AND THE RAIN DIAL

JOHN FOAD

On a visit to Scotney Castle, I was struck by the beauty of the horizontal dial by Dudley Adams (Register Number 3301). However, the morning's rain lay on the flat brass dial plate, and it was difficult to appreciate the detail or even to read the time. It made me wonder why the reclining dial, close to the horizontal, is not used more often in the garden. It keeps the surface clear of rain, reduces the effect of weathering, adds an element of interest to the design, and is easier to read.

If William of Occam were alive today, and a member of the Society, I am sure he would object to the unnecessary multiplication of terminology, but I would like to suggest that such a dial should be recognised as a type, and called a Laid Dial, with the gradient being called the Angle of Lie (AOL) (see Figure 1). The reasoning behind the name is that the dial is 'laid' on a pedestal, or indeed on the ground, in much the same way as one might lay a book on a lectern. I must admit I also like the palindrome! In these latitudes, it would be natural to raise the Noon end of the dial, and by convention I would call this a positive AOL. A negative angle becomes more appropriate as one nears the equator. Where the AOL reaches the angle of latitude, we have a Polar Dial; and when the (negative) angle is equal to the co-latitude, it becomes an Equinoctial or Equatorial; but I would reserve the term Laid Dial to describe only those where the AOL is less than say 30°.

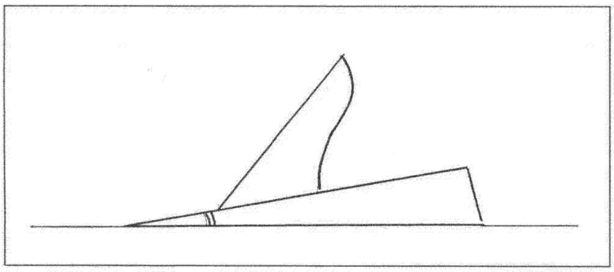

Fig. 1. Elevation of a Laid Dial, looking West. The marked angle is the Angle of Lie.

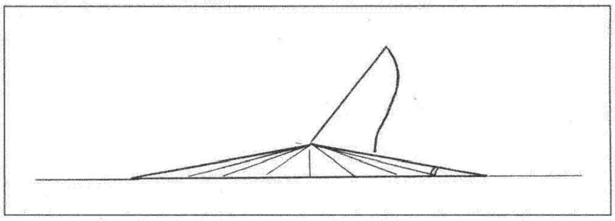

Fig. 2. A Rain Dial, type 1. The base is a shallow circular cone, with the marked angle being the Rain Angle.

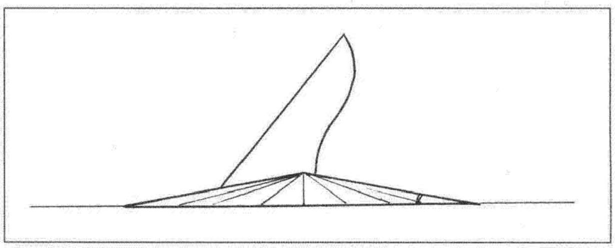

Fig. 3. Rain Dial, type 2. The gnomon is offset, using the surface better than type 1, but requiring curved hour lines (not shown)

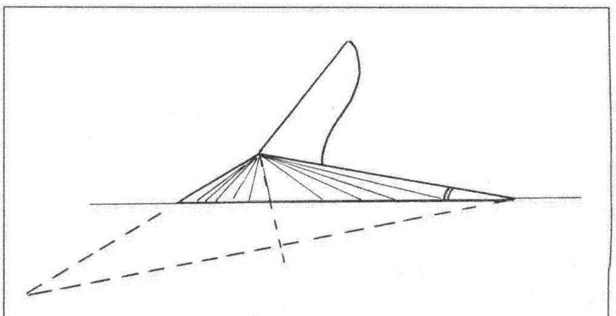

Fig. 4. Rain Dial, type 3. The cone is now tilted, to allow better positioning of the gnomon than in type 1. Again the marked angle is the Rain Angle. The dial plate is elliptical in plan view, with straight hour lines.

There are of course many approaches to the problem of avoiding the rain lying on a horizontal dial plate. Another would be to raise the centre, or some internal portion, of the plate to allow drainage, and this I would call a Rain Dial.

The name describes the purpose of the form, with a pleasing note of paradox, and still has some of the symmetry of the name 'Laid Dial'. A simple version would use the surface of a shallow right circular cone (Fig 2). The Rain Angle would be defined as the complement of the semi-vertex angle of the cone. Nearer the Pole, a negative Rain Angle would be best, but this would definitely need a drain hole at the vertex! The Rain Dial could take many forms. If the axis of the cone is vertical, and the gnomon passes through the vertex as in Fig 2, the hour lines are straight and I think even I could make one. But this would not be an attractive design, nor efficient in the use of the surface of the dial plate. With the vertex in the centre of the plate, and the gnomon offset to the South as is usual (Fig 3), I believe the lines become hyperbolae. This could make a very attractive dial, but I leave the construction as an exercise for more capable members! The best form might be with the axis of the cone tilted towards the South, and the gnomon through the vertex (Fig 4). This would give a pleasing elliptical plan view, and would retain straight hour lines, and spread them well. The Rain Angle would be defined as the gradient at the Noon end of the dial, as marked in the figure. A full specification of this form would then need to include the semi-vertex angle of the cone.

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FINDING A NORTH-SOUTH LINE

W. S. MADDUX

As used by dialists, the word Azimuth means the absolute horizontal angle of the sun relative to true South. To remove ambiguity, its direction must be explicitly designated as east or west from the local meridian's true south direction. The term Bearing may here be thought of as an unambiguous direction referred to a horizontal circle of 360°, with 0° at true North, 90° at East, 180° South, and so on.

The arrangement described first below for obtaining the N-S line (with the observer sunward of the plumb line) can be precise enough for aligning just about any ordinary sundial. The second (with the observer shadow-ward of the plumb line) can, with care, yield precision within a fraction of a minute-of-arc, which is beyond that needed for almost any practical sundialing purpose.

I - OBSERVER IS SUNWARD:

Fig. 1 illustrates a technique I have often used for observer's-back-to-sun determinations of the vertical plane of the sun's position. It especially helps when dealing with shadows cast by quite high or by quite low sun elevations. It is also of advantage in situations where the ground surface is not level and/or is not planar.

Preferably, the plumb line would be suspended from a pre-existing, "permanent" support; but if need be, one can arrange a tripod or other temporary setup from which to hang the line. (The suspension "pulley" may be a screw hook, a bent nail, or the like, such that it will permit the bob to move freely up and down as the vertical string-length is adjusted. A hook is here better than a closed eye, as it more conveniently allows for removing and re-hanging the

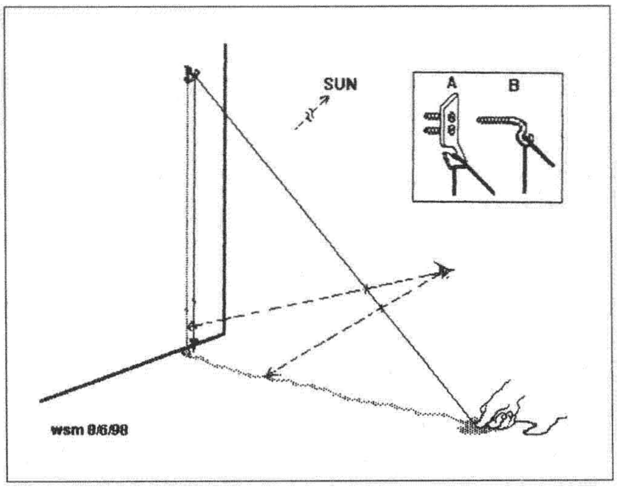

Fig. 1. Observer to sunward

plumb line (during installation of a dial, for instance) Sketches "A" and "B" of the inset in Fig.1 show two sample variations for a hook's form. Since it is desirable to minimise horizontal displacement of the effective suspension point as the slanted string-part is being adjusted in azimuth, the lower part of the hook is best formed as a fairly narrow constraining vee, but it should have its edges smoothly rounded to allow the string to slide evenly, without "stick-slip" hesitations.

The length of string from hook to hand is whatever you find convenient, and line may be paid out or taken in, to adjust the weight a little above the ground as you seek a place for marking, right up until the timed moment of observation. (To arrest any swinging of the bob, lower it to contact the ground, then gently drag it upward until it hangs free. Repeat as necessary.)

It helps if the supporting structure provides a vertical backing-surface to receive the shadows of the string, but you can alternatively use the shadows cast upon the ground. (If shadow contrast is a problem, place a light-toned card or other panel on wall or ground as needed.) Keep adjusting the string's free end to maintain shadows of both parts of the string superposed, and record its position with a south-end mark upon a stake or batter board at the proper time. At that moment, both string-parts and their respective shadows are in a common (vertical) plane with the sun, and if you position your eye directly behind the slanted string, all are seen superimposed. I sometimes like to move to one side and closer to the weight, while reaching back with one hand to control the string's direction. That way, my own shadow doesn't interfere, and I can lean closer to the two string-parts' shadows to gauge their coincident alignment. I usually tie a figure-of-eight knot a few inches above the weight, to help to distinguish the two merging shadows for correct sense feedback-to-null whilst adjusting the slanted string's bearing. Such a knot is symmetrical about its own string-part, so in order to have the combined shadows also appearing symmetrical re the knot, the movable slanted string-part's shadow must be placed on-center with the vertical string-part and its shadow.

If you have not already done so, you should fix a north-end marker centered directly beneath the suspended plumb bob.

Of course you can later recover the observed direction at any time, by putting the string back on the hook and upon the south-end mark you have made. You may then use the paired string-parts as "sights," in order to project and mark intermediate in-line points along the ground's intersection with the defined vertical plane.

II - OBSERVER IS NORTHWARD:

For observer-north measurements, as in Fig. 2, a support, (gibbet, tripod, roof eave, or the like,) has been used to suspend a plumb line, and to place a mark on a stake or other fixed anchorage, placed by plumb-line directly below the suspension point Sp. A string fastened at this lower mark Lo takes a more-or-less horizontal line L northward toward a batten board or similar fixture. At the pre-calculated watch time for local apparent noon, the shadow of an upper stretch of the plumb line (as represented by the half-tone line S) intersects the batten board at Lt, where the taut horizontal string may be compared - and symmetrically aligned with - the shadow, in order to establish and to mark the direction of line L. Of course the entire plumb line casts a vertical planar shadow which at true noon corresponds to part of the meridian plane. For purposes of illustration, only a slender linear "shadow-ray" segment within that shadow plane is indicated by the half-tone line S.

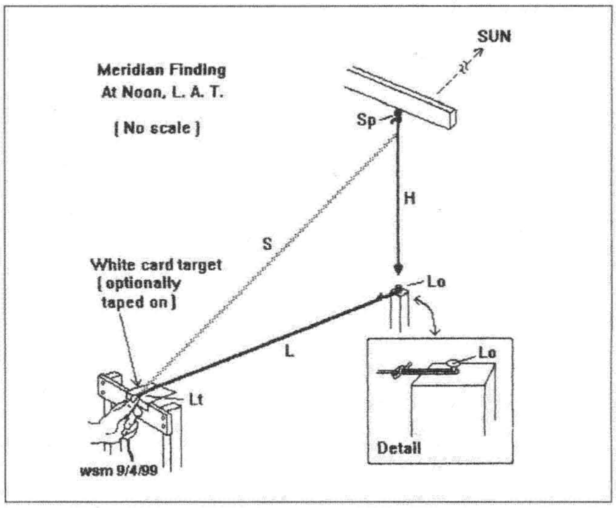

Fig. 2. Observer to northward

The string L, and plumb line H's shadow, can be brought to symmetric coincidence within the best resolution of the observer's near point vision, to record the direction of a fairly long baseline L. For example, if the string L can be centered within + / - 0.2 mm under the plumb line's shadow S, and if L is two meters long, the estimated maximum resulting error-contribution in angle will be 0.2 ÷ 2000, or 0.0001 radian. Multiplying by 180° /pi, and again by 60 to convert to minutes-of-arc, we find an equivalent 0.34 minute-of-arc estimated limit of uncertainty in the recorded direction of Lo. A small notch cut into the northward upper edge of the batten board will allow the line L to be re-established by a taut string for later use. The plumb line and its suspension support are no longer required, once the two ends of L have been fixed. The realised dimensions in a particular case will generally differ from those of the example cited above, but the favorable circumstances of a relatively long baseline, as established by close-up viewing of the determining compared lines' coincidence (H's shadow and string L) should still obtain. Precision of a fraction of a minute-of-arc may reasonably be expected under quite practicable observational arrangements.

Although the above discussion assumed a noon observation, perhaps the noon elevation of the sun may present an inconveniently large height to length ratio, H/L. In such circumstance, one might choose to employ a less unwieldy support from which to suspend H, and to deploy the observational setup for an arbitrary known time (and predicted bearing) several hours before or after noon. Again, simple plane trigonometry can then be applied - and a steel tape used -to lay down side-lengths of a horizontal triangle, in order to mark out the true meridian's bearing relative to the known direction found by the observation at the lower-than-noon solar altitude.

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THE STEREOGRAPHIC PROJECTION AS A GRAPHICAL METHOD FOR DESIGNING SUNDIALS PART 1: HORIZONTAL AND SOUTH FACING VERTICAL SUNDIALS

TONY BELK

1. INTRODUCTION

The design of sundials by graphical means has a long history. The methods work well, they allow those who do not wish to do multiple trigonometrical calculations to design sundials but they often do not allow a clear visualisation of the operation of sundials. Their careful use allows a reasonable level of accuracy but not the precision of calculation. However the results of calculation have to be transferred graphically to make a dial. The stereographic projection is a method used for centuries by geographers to map the surface of the globe, and more recently by crystallographers to represent the three dimensional arrangement of directions and planes. The sun's direction changes regularly and predictably and is known at all hours and dates in the year and can be represented clearly on a stereographic projection. This construction not only allows the graphical design of any sundial using a flat dial plate at any inclination for any location in the world, but also enables a clear visualisation and understanding of a sundial's design and operation.

2. THE STEREOGRAPHIC PROJECTION.

The stereographic projection is a means of plotting features on the surface of a hemisphere onto a flat surface. It has been in use by geographers and astronomers since the second century **AD** and more recently by crystallographers. Any feature on the hemisphere's surface is projected onto the horizontal equatorial plane from the opposite pole. Important features of the projection are that a circular arc on the hemisphere becomes a circular arc on the projection and the angle between two lines on the surface of the hemisphere is preserved as the same angle in the projection. Crystallographers use the projection to represent directions and planes in crystals and to measure angles between planes and between directions. Because of its ease of representing the three dimensional disposition of planes and directions in two dimensions it can also be used as a universal graphical method for designing any planar sundial for any location. All features on the hemisphere's surface are projected onto the horizontal equatorial plane from the opposite pole **O** as shown in fig 1. A direction through the centre of the hemisphere cuts its surface in a point **d** and that direction appears on the projection as point **p**. A plane through the centre of the hemisphere cuts the surface of the hemisphere in a semicircular arc which is a great circle of the hemisphere. That arc is projected as shown in fig 1 as an arc of a circle subtended by a diameter of the projection. Any circular arc subtended by a diameter of the projection is a great circle, even a diameter of the projection.

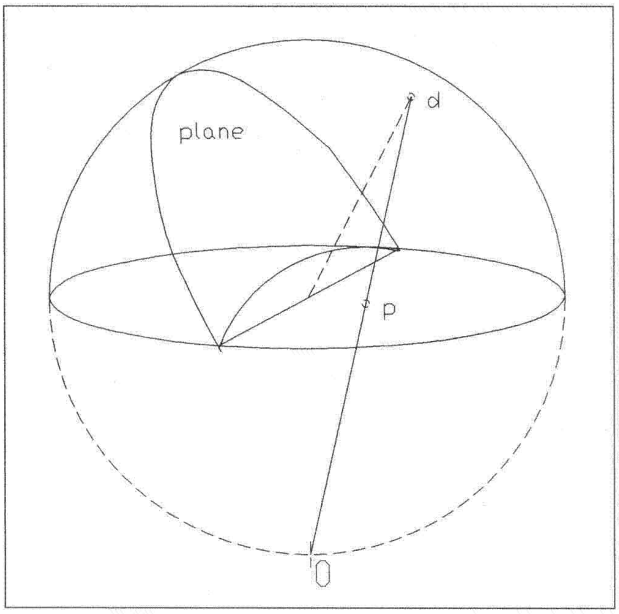

Fig. 1. Representation of a point and a plane on the stereographic projection.

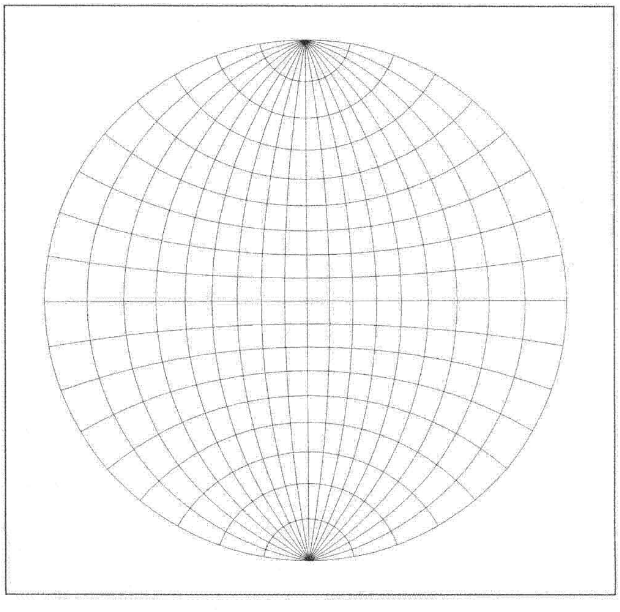

Fig. 2. Wulff net with 10 degree intervals used for measuring angles on the stereographic projection.

Angles on the projection are measured with a special type of protractor known as a Wulff net, fig 2. This is a plot of the lines of latitude and longitude produced by the method shown in fig 1, in this case at intervals of 10 degrees to illustrate the construction. A Wulff net used for measurements would have intervals of only 2 degrees. The Wulff net enables the angle between any two directions or any two planes to be measured, the plane containing any two directions to be drawn, and the direction of intersection of any two planes to be found. It also enables the projection to be rotated about any axis. To measure the angle between two directions the Wulff net must be rotated about its centre until a great circle is found on which they both lie and the angle can then be read round the great circle. Notice that the lines of equal latitude on the Wulff net are not great circles because they are not subtended by a diameter. They are small circles and are circular arcs linking all directions making the same angle with the top to bottom axis of the net.

In using the stereographic projection to design sundials the direction of the sun at a particular time and date becomes a point on the projection and the plane on which the shadow is cast is a great circle. It can be used to design any type of sundial with a flat dial plate for any latitude in the northern or southern hemisphere. The angles of the hour lines, the position of the equinox and solstice lines and the time of sunrise and sunset for any declination of the sun can all be found graphically.

3. HORIZONTAL AND SOUTH FACING VERTICAL DIALS.

To design sundials graphically one can use a stereographic projection with the sun's directions projected on the polar plane defined by the east and west compass points and the celestial pole. If the directions of the sun are plotted on a stereographic projection in this way for each hour throughout the year the result is shown in fig 3. All directions between **E** and **W**, above the winter solstice arc and below the summer solstice arc are possible directions for the sun. The plane of the projection is that containing the north celestial pole C and the east west axis **E, W**. This projection allows the changes in sun's direction to be visualised and followed and allows any plane, horizontal, vertical or at any other angle to drawn and the sundial on that plane to be drawn. The hour lines on the projection are great circles 15 degrees apart. Smaller time increments could be included by drawing more great circles with smaller angular separations. Equally any angle of sun's declination can be plotted between the solstice and equinox lines. Intermediate declinations and time intervals have been omitted from fig 3 for simplicity.

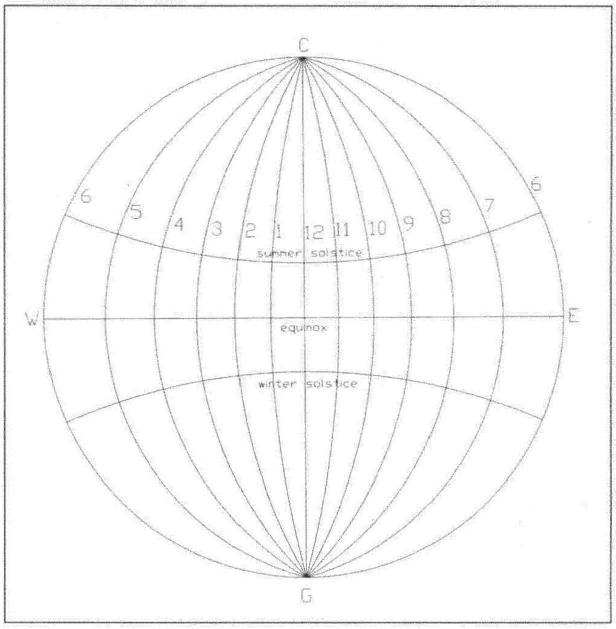

*Fig. 3. Stereographic projection of the sun's direction through the day and year projected onto the polar plane with **C** the celestial pole.*

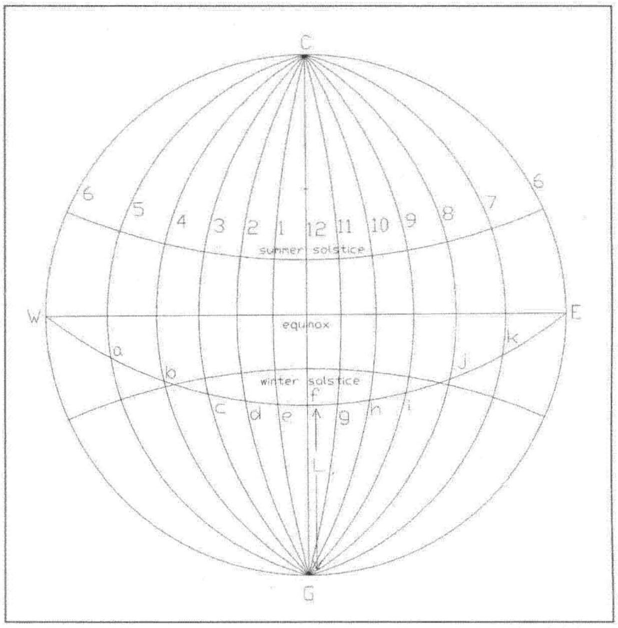

*Fig. 4. The horizontal plane at latitude **L** superimposed on fig 3.*

To design a horizontal dial for latitude **L** the horizontal plane at latitude **L** must be drawn on the projection. The method of using fig 3 and the Wulff net to design a dial is described in Appendix 1 below. The horizontal plane at latitude **L** is the great circle that cuts the vertical axis L degrees from the south celestial pole **G** of the projection, fig 4. The great circle **W, a, b, c, d.....E** is the projection of the horizontal plane at latitude **L**. The gnomon direction is **C**. The angle between **W** and **a** is the angle between the hour lines for 5 and 6. Similarly the angle between **a** and **b** is the angle between the hour lines for 4 and 5. The angles

W-a, a-b, b-c etc must be read from the projection using a Wulff net. The angles are listed below for the case illustrated in fig 4. Also for the case illustrated in fig 4 at the winter solstice the sun rises when the projection of the plane crosses the winter solstice line and sets when they cross again, i.e. at about 8.15 am and 3.45 pm.

TABLE 1

Hour lines	Angle	Cumulative angle
6 – 7	19	19
7 – 8	17	36
8 – 9	16	52
9 – 10	14	66
10 – 11	12	78
11 – 12	12	90
12 – 1	12	102
1 – 2	12	114
2 – 3	14	128
3 – 4	16	144
4 – 5	17	161
5 – 6	19	180

Fig 4 also allows the positions of the solstice and equinox lines, or indeed any other line of equal declination, to be drawn. The angles **5-a, 4-b** etc are the angles between the hour line on the dial plate and the sun's direction at summer solstice. These enable the position of the summer solstice line to be drawn on the dial if the height of the nodus is known. Similarly the angles between **a, b, c,** etc and the equinox directions gives the position of the equinox line on the dial, and the angles between **c, d, e, f, g, h,** and I and the winter solstice directions gives the winter solstice line. The angles read from fig 4 are tabulated below.

TABLE 2

Hour	Summer	Equinox	Winter
12	62	38.5	15.5
11 & 1	61	37.5	14
10 & 2	58	34.5	11
9 & 3	52.5	29	5.5
8 & 4	45.5	22	
7 & 5	35.5	12	
6	23.5	0	

The angles taken from fig 4 have been used to draw the horizontal dial fig 5 using the construction described in Appendix 2 below. The gnomon is drawn to scale and the nodus is the top point of the gnomon.

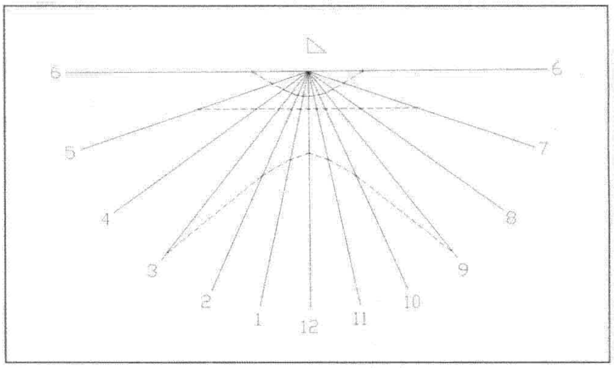

Fig. 5. The horizontal sundial constructed from the angles read from fig 4.

To design a south facing vertical dial for latitude **L** a similar construction is required, but now the vertical plane must be drawn which cuts the vertical axis of the projection at angle **L** above the centre of the projection, fig 6. The horizontal plane is also shown dashed in fig 6. As for the horizontal dial the gnomon direction is **C**, and the angles of the hour lines are read around the great circle representing the vertical plane. The direction **P** is the vertical direction. In fig 6 at the summer solstice the sun shines on the south facing dial whenever the projection of the vertical plane is above the summer solstice line i.e. from about 7.25 am to 4.35 pm. Also as for the horizontal dial the solstice and equinox lines can be marked by reading the angle around the hour line from the vertical plane to the equinox or solstice lines.

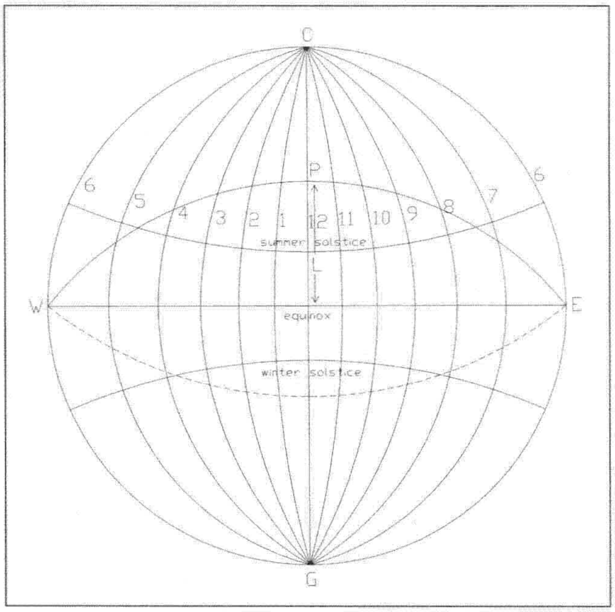

*Fig. 6. The vertical south facing plane at latitude **L** superimposed on fig 3 with the horizontal plane at latitude **L** shown dashed.*

All the above discussion has assumed that the dial is for the northern hemisphere. In the southern hemisphere exactly the same constructions hold but a south facing vertical dial for latitude **L** in the northern hemisphere is the same as a horizontal dial for latitude **90-L** in the southern hemisphere. Similarly a horizontal dial for latitude **L** in the northern hemisphere is the same as a north facing vertical dial for latitude **90-L** in the southern hemisphere.

For all planar dials if the direction of the celestial north pole is pointing out of the dial face, the hour lines are arranged in a clockwise sequence. If the direction of the celestial north pole is pointing into the dial face, the sequence of the hour lines is anticlockwise. This rule works for any type of planar dial in any location on the globe.

APPENDIX 1

To use the stereographic projection to design a sundial the best method is to have the projection fig 3 and the Wulff net fig 2 on card with drawing pins sticking upwards through the cards at their centres. A piece of tracing paper can be placed central on the Wulff net with the pin sticking through it so that it can be rotated about the centre of the Wulff net. First draw the horizontal plane at the latitude required by tracing it from the correct great circle of the Wulff net. Then draw the plane of the dial face again by tracing the correct great circle from the Wulff net. Now remove the tracing paper and place it on the projection so that it can rotate about the centre. Align the horizontal plane by rotating the tracing paper. Now the angles between the hour lines and between the dial face and the sun's declination can be marked on the tracing paper. Replace the tracing paper on the Wulff net to measure the hour line angles and the equinox and solstice mark angles around a great circle of the Wulff net.

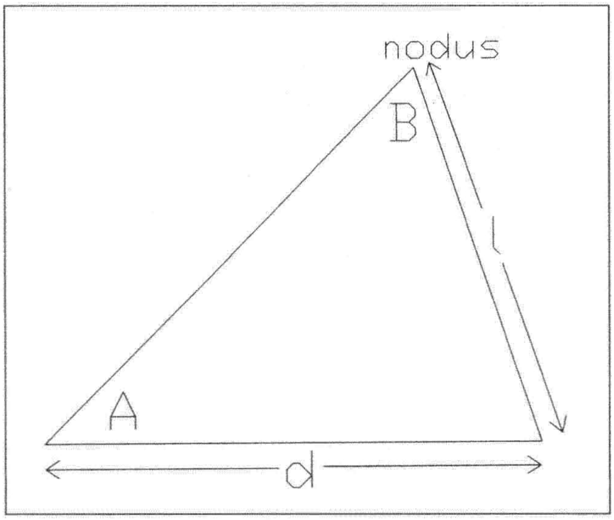

Fig. 7. Construction for determining the position of equinox and solstice marks on a sundial.

APPENDIX 2

The graphical method for determining the positions of the solstice and equinox marks is illustrated in fig 7. For example, to determine the positions of the solstice and equinox marks on the hour line for 3, construct a triangle such that angle **B** is 90-declination or 66.5 for the summer solstice. The length l is the sloping height of the gnomon to the nodus. The angle **A** is the angle **3-c**. The distance **d** is then the distance from the root of the gnomon along the hour line 3 to the summer solstice mark. Similar constructions are used to position the equinox and winter solstice marks, and for all other hour lines. The same construction can be used to mark the solstice and equinox lines on a vertical dial.

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The stereographic projection is introduced in many university level text books such as C S Barrett and T B Massalski, *Structure of Metals*, McGraw-Hill, 1966 or J R Moon, *Introduction to the Stereographic Projection* The Institute of Materials, 1972. The International Union of Crystallography web site also has a good introduction to the projection and the use of Wulff nets; www.iucr.ac.uk.

REFERENCES

The only references to the use of the stereographic projection in the graphical design of sundials appear to be those concerning William Oughtred (1575-1660).

He wrote in 1598 *An Easy Way of Delineating Dials by Geometry* which was published in English in his *Clavis Mathematicae* in 1647. He also wrote a tract on drawing a dial on any plane surface however inclined, which was published with *Circles of Proportion* in 1632. In the tract he described an instrument, which he later called the horizontal instrument. This is a stereographic projection of the sun's directions throughout the day and year projected onto the horizontal plane at the latitude of interest. This allowed the direction of the sun to be determined graphically at that latitude and also made possible the delineation of a dial on any plane surface of whatever inclination at that latitude. He did not however indicate how to mark the solstice or equinox lines. Oughtred also used his horizontal instrument in the design of double horizontal sundials:

F. Sawyer: 'William Oughtred's Double Horizontal Dial' *Compendium, Journ. NASS.* **4**, (1), 1-5 (1997)

W. Oughtred: 'The Description and Use of the Double Horizontall Dyall' *Compendium, Journ. NASS.* **4**, (1), 6-11 (1997)

This application of the projection to the graphical design of sundials will be covered in the second part of this article

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